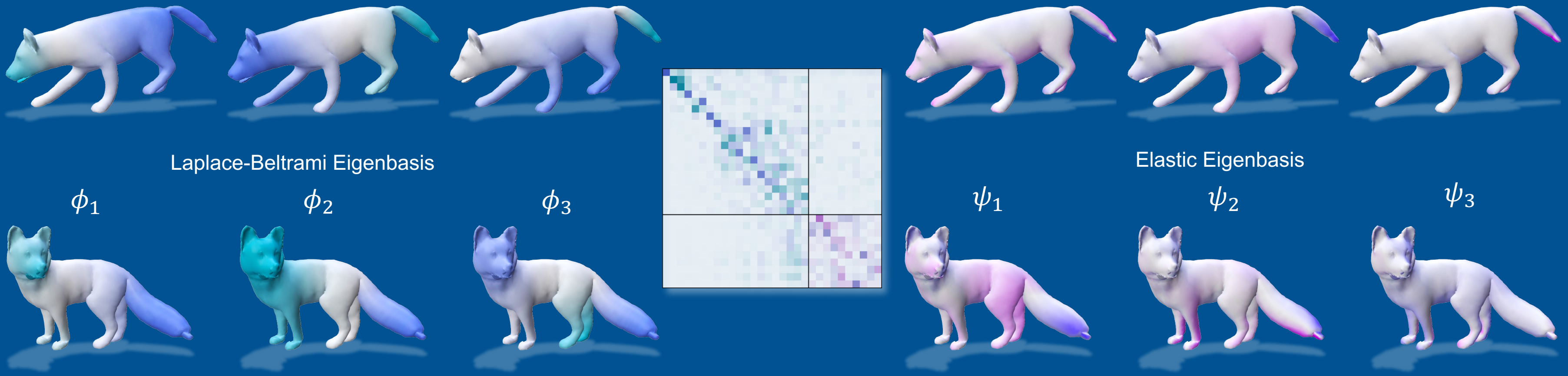
Hybrid Functional Maps

for Crease-Aware Non-Isometric Shape Matching

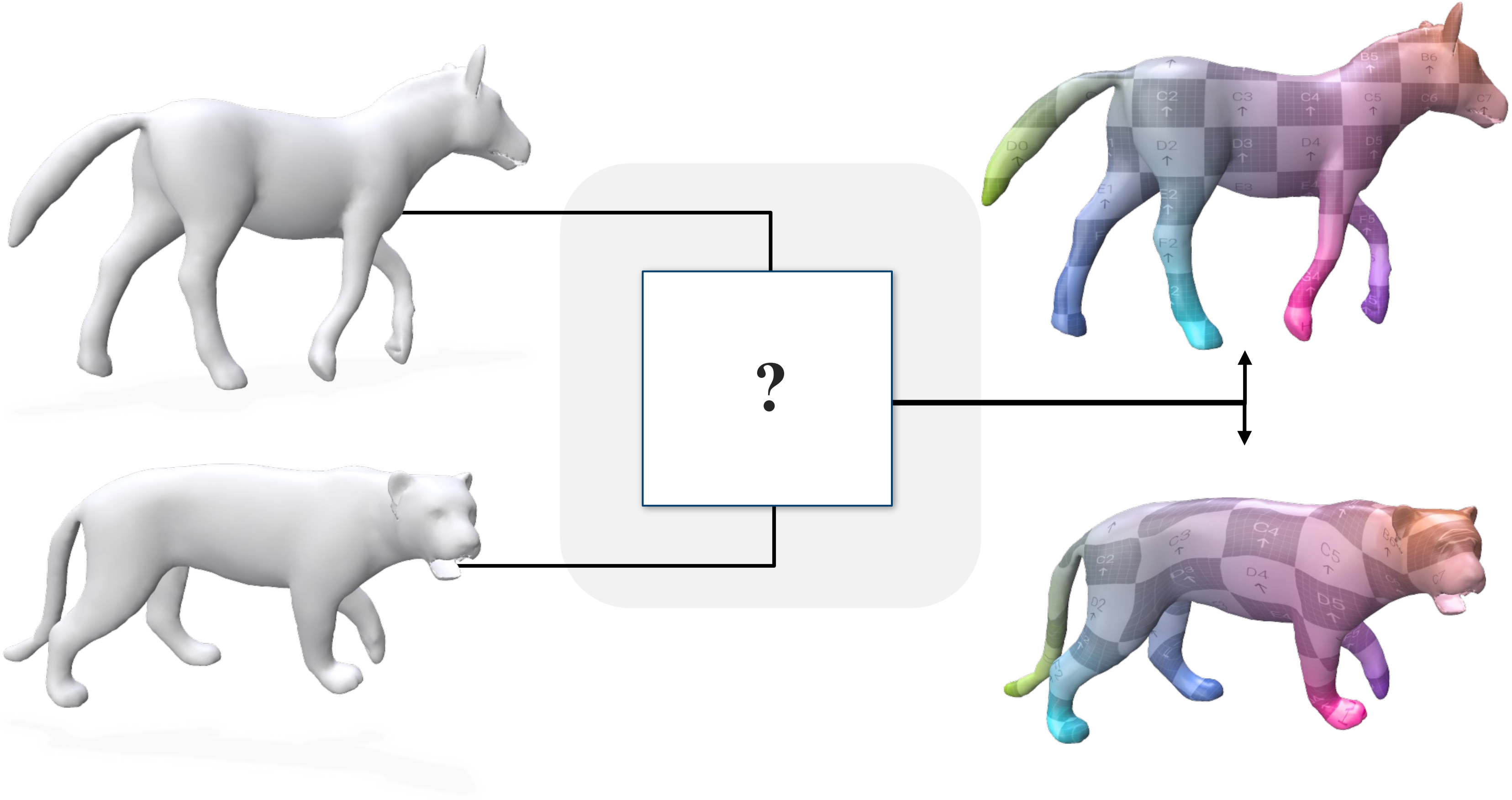


Lennart Bastian* Yizheng Xie* Nassir Navab Zorah Lähner

Bastian, L., Xie, Y., Navab, N., & Lähner, Z. (2023). Hybrid Functional Maps for Crease-Aware Non-Isometric Shape Matching. *arXiv preprint arXiv:2312.03678*.

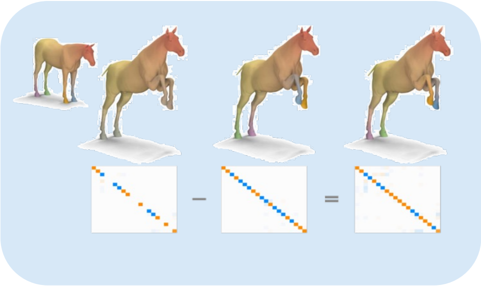
Goal: Non-isometric Shape Matching

Input Shapes

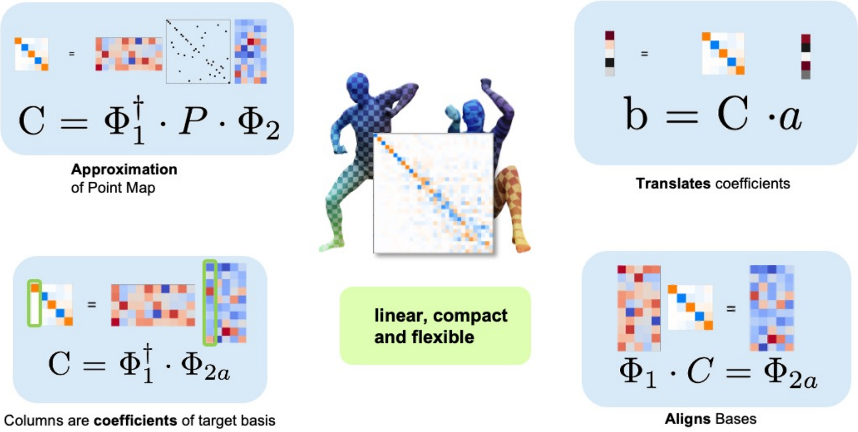


Output Dense Correspondences

Background Work



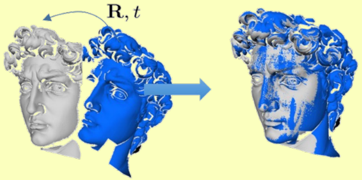
Functional Maps [Ovsjanikov et al. 2012]



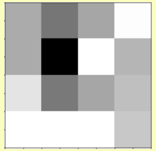
Ovsjanikov, M., Ben-Chen, M., Solomon, J., Butscher, A., & Guibas, L. (2012). Functional maps: a flexible representation of maps between shapes. ACM Transactions on Graphics (ToG), 31(4), 1-11.
Sharp, N., Attaiki, S., Crane, K., & Ovsjanikov, M. (2022). Diffusionnet: Discretization agnostic learning on surfaces. ACM Transactions on Graphics (TOG), 41(3), 1-16.

Rigid Alignment

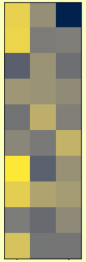
Rigid

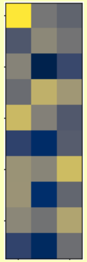


4x4 Rt



aligns
xyz
coordinates

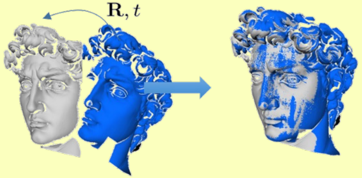




Ovsjanikov, M., Corman, E., Bronstein, M., Rodolà, E., Ben-Chen, M., Guibas, L., ... & Bronstein, A. (2016). Computing and processing correspondences with functional maps. In *SIGGRAPH ASIA 2016 Courses* (pp. 1-60).

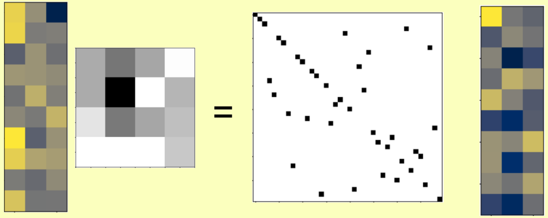
Rigid Alignment

Rigid



4x4 Rt

aligns
xyz
coordinates



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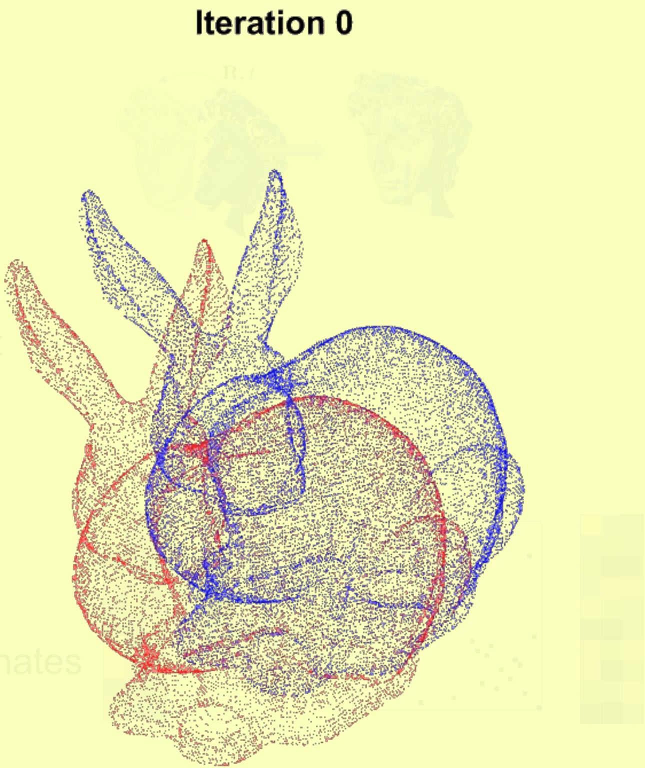
Rigid Alignment

Rigid

4x4 Rt

aligns
xyz
coordinates



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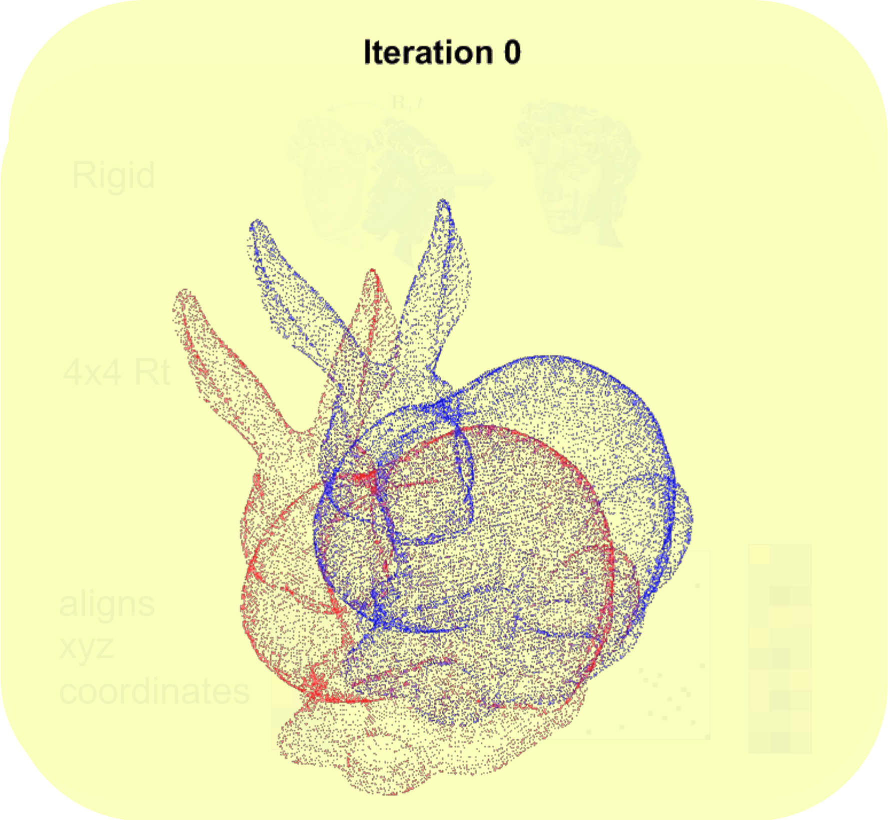
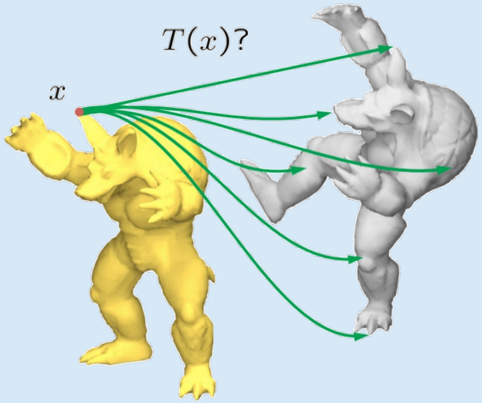
Rigid Alignment

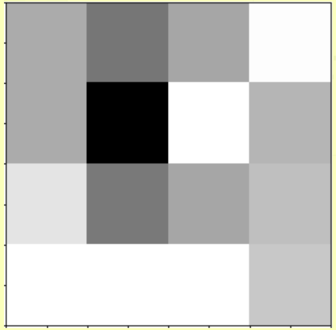




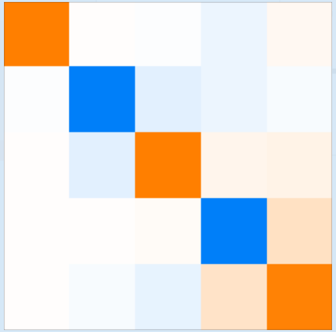
Non-rigid, can we have a similarly compact representation?

Ovsjanikov, M., Corman, E., Bronstein, M., Rodolà, E., Ben-Chen, M., Guibas, L., ... & Bronstein, A. (2016). Computing and processing correspondences with functional maps. In *SIGGRAPH ASIA 2016 Courses* (pp. 1-60).

Compact Representation



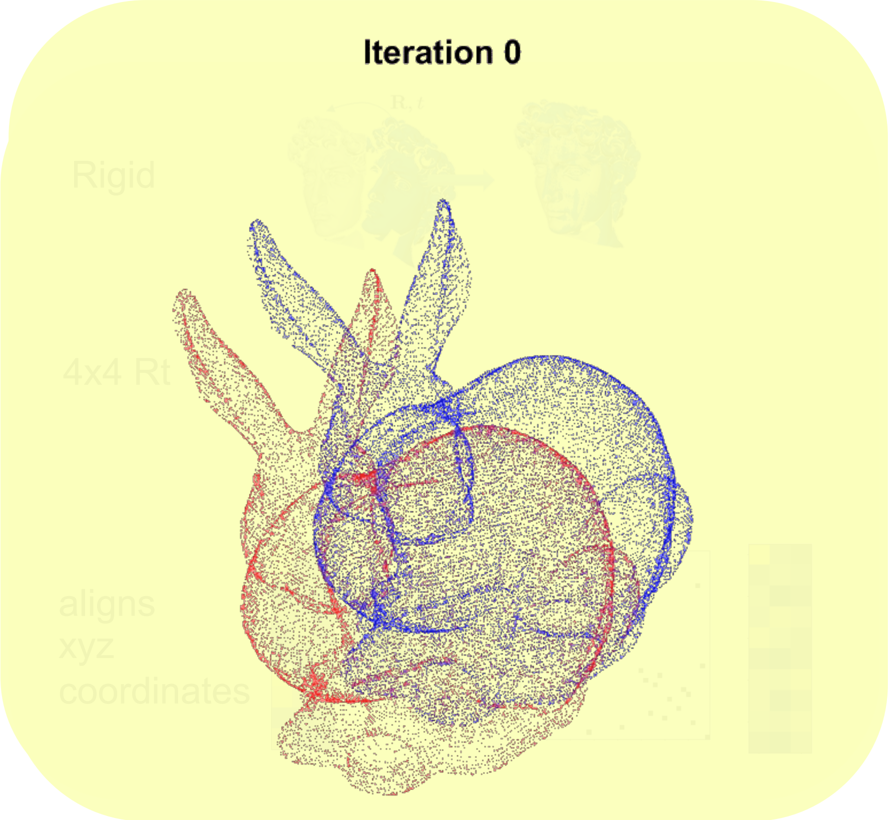
Rigid

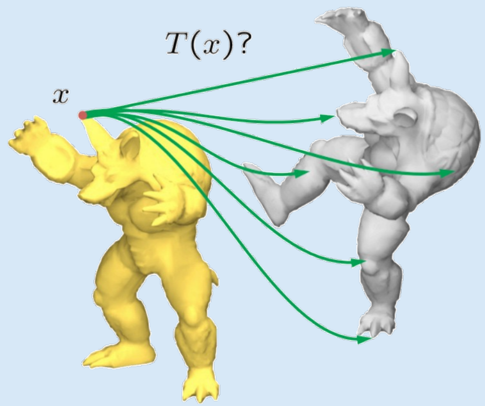


Non-Rigid

Ovsjanikov, M., Corman, E., Bronstein, M., Rodolà, E., Ben-Chen, M., Guibas, L., ... & Bronstein, A. (2016). Computing and processing correspondences with functional maps. In *SIGGRAPH ASIA 2016 Courses* (pp. 1-60).

Rigid Alignment

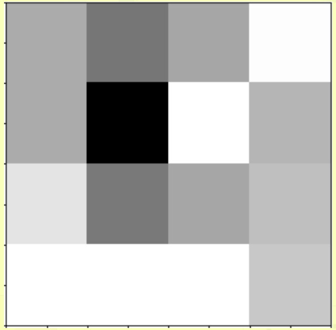




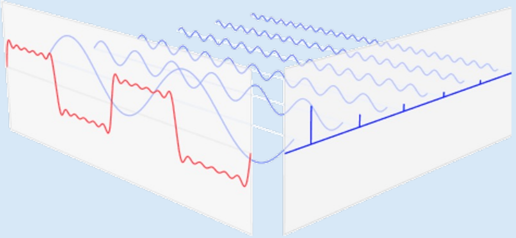
Point to Point, NP hard Problem

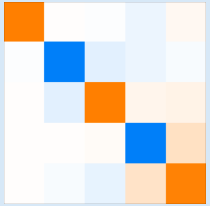
Ovsjanikov, M., Corman, E., Bronstein, M., Rodolà, E., Ben-Chen, M., Guibas, L., ... & Bronstein, A. (2016). Computing and processing correspondences with functional maps. In *SIGGRAPH ASIA 2016 Courses* (pp. 1-60).

Spectral Rigid Alignment



Rigid Alignment





Functional Map

Ovsjanikov, M., Corman, E., Bronstein, M., Rodolà, E., Ben-Chen, M., Guibas, L., ... & Bronstein, A. (2016). Computing and processing correspondences with functional maps. In *SIGGRAPH ASIA 2016 Courses* (pp. 1-60).

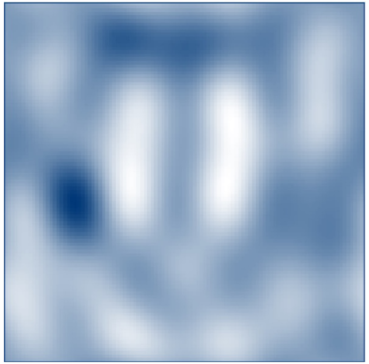
Fourier Image analysis:

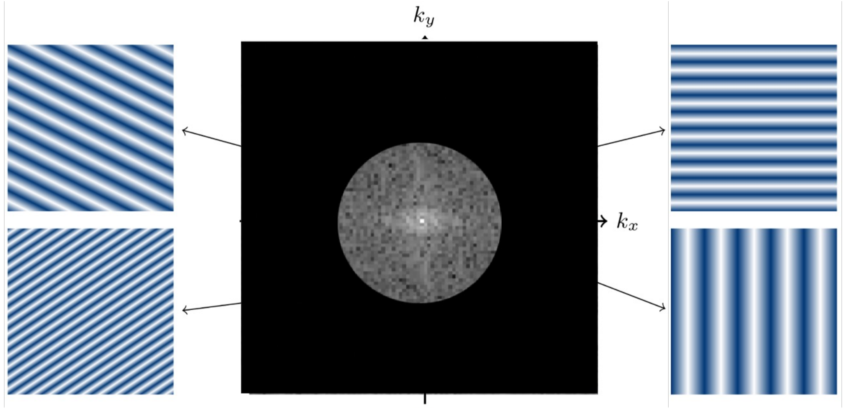
Discretized 2D Grid / Image

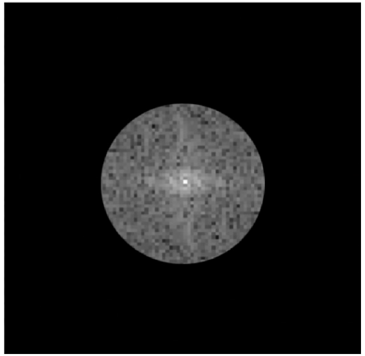
Rippel, O., Snoek, J., & Adams, R. P. (2015). Spectral representations for convolutional neural networks. *Advances in neural information processing systems*, *28*.

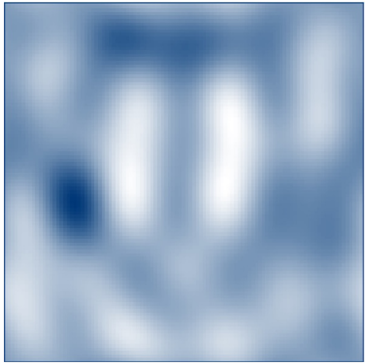
Image compression:

Truncated coefficients to only low frequency

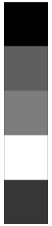


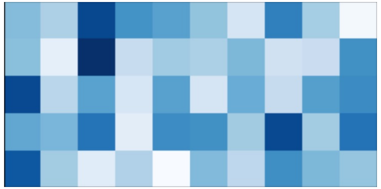


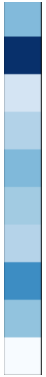




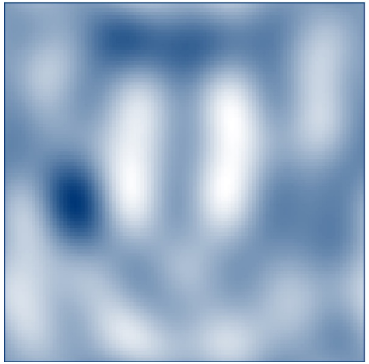
$$a = \Phi^{-1} \cdot f$$

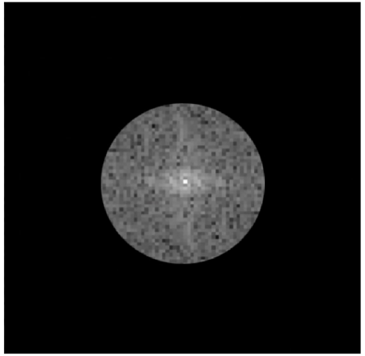


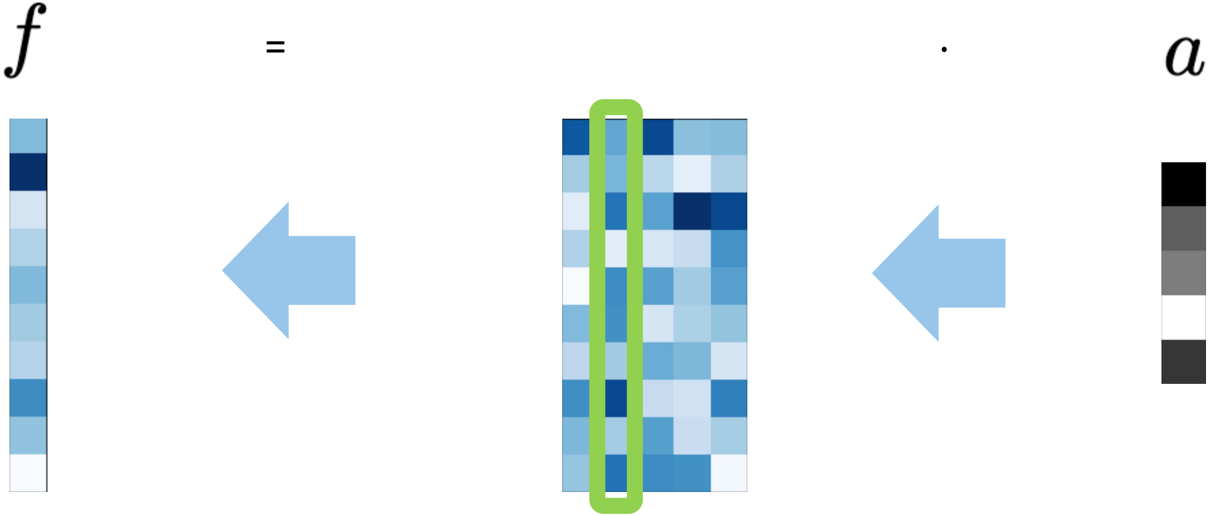




From an image, project to spectral coefficients





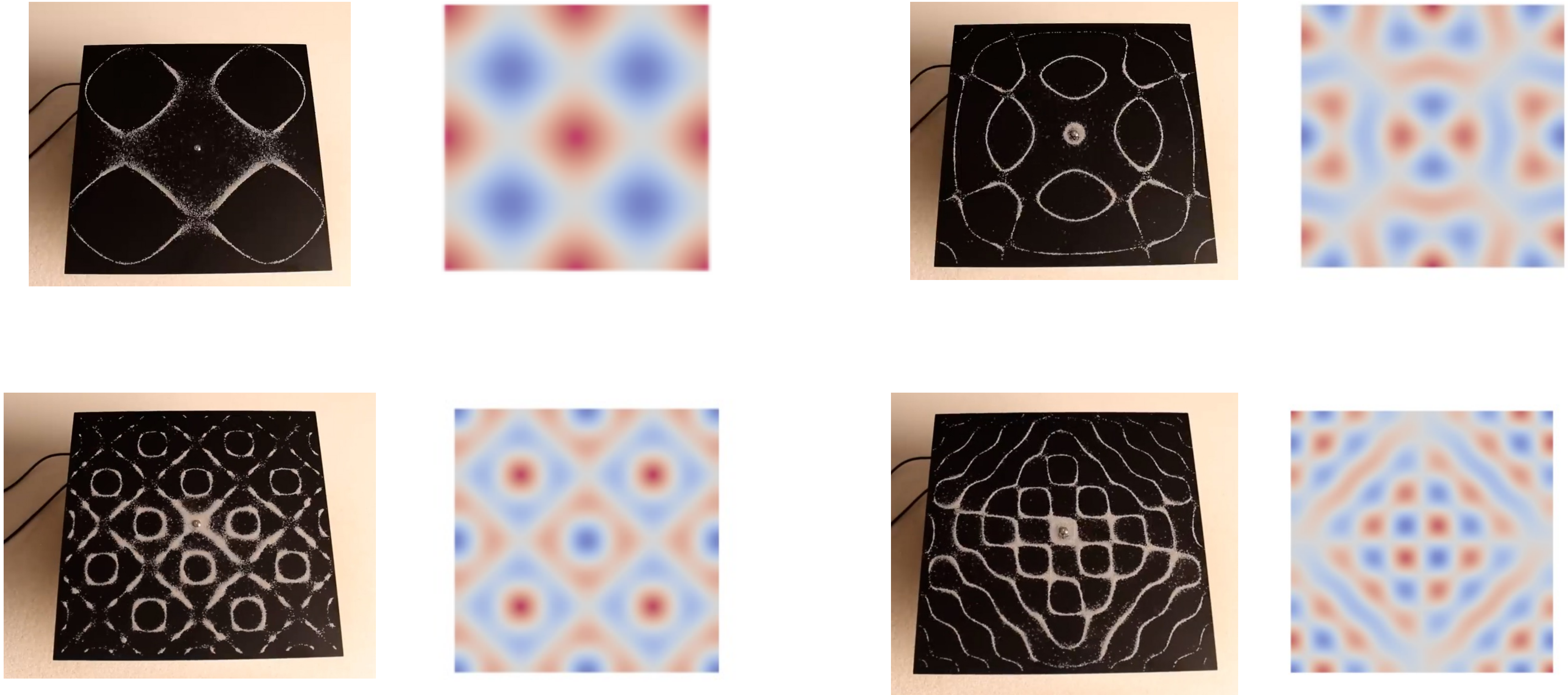


Each column vector represents an entire image

Fourier Basis Functions are **orthogonal**

From coefficients to reconstructed image

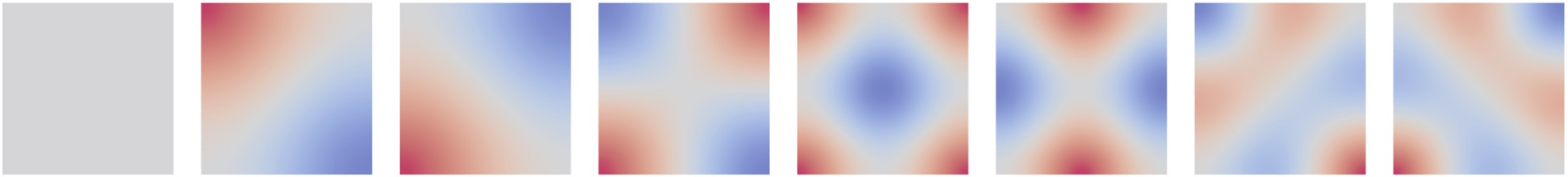

345 hz
https://youtu.be/wvJAgrUBF4w
youtube.com/brusspup

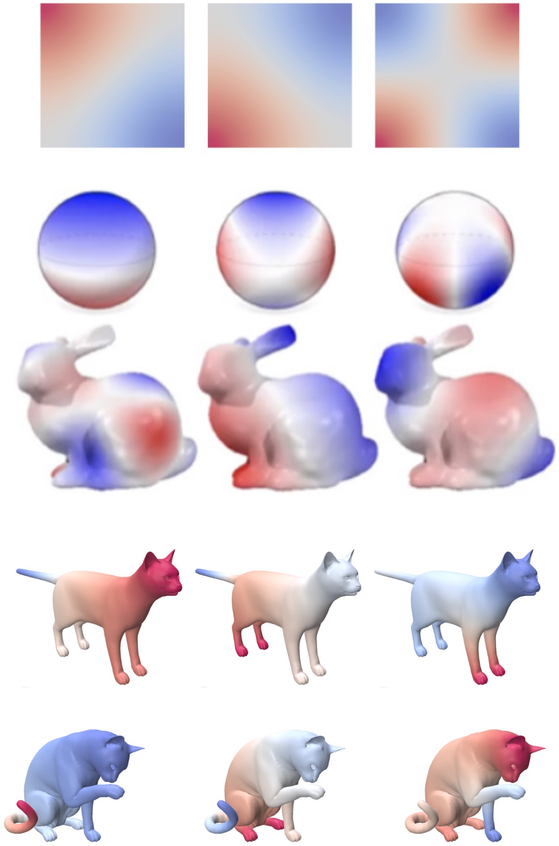


Chladni plate patterns

https://youtu.be/wvJAgrUBF4w

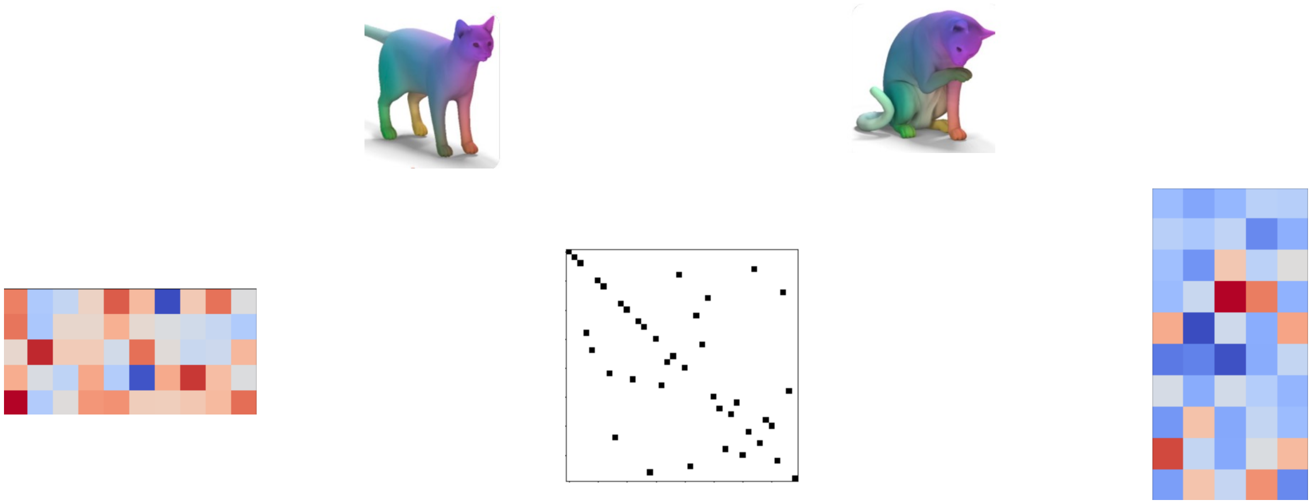
Eigenfunctions of the Laplace-Beltrami Operator

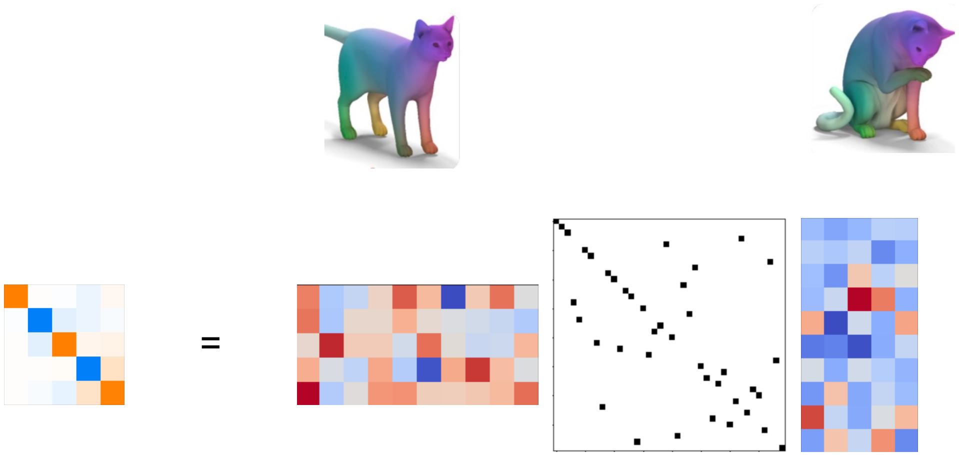




LBO Basis functions are defined for any shape surface

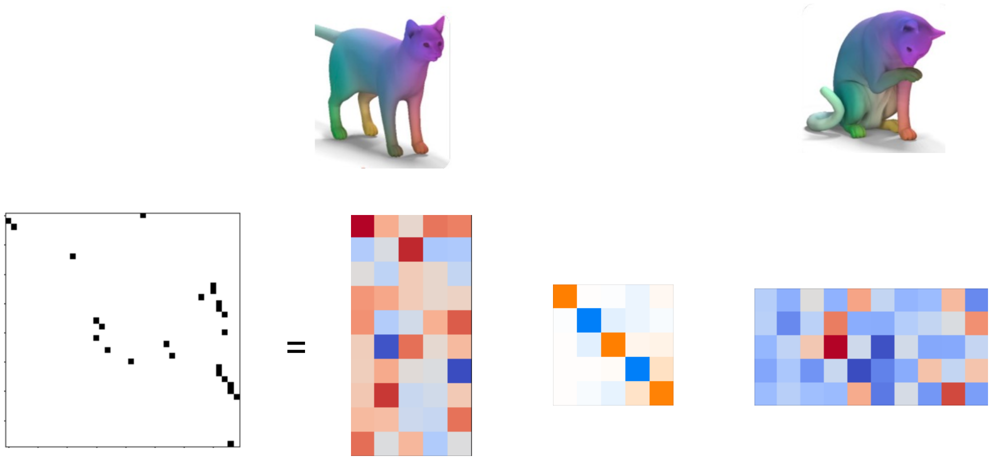
https://brickisland.net/DDGSpring2021/2021/04/20/lecture-18-the-laplace-beltrami-operator/





$$C = \Phi_1^{\dagger} \cdot P \cdot \Phi_2$$

A functional map is a rank-k approximation of a point map



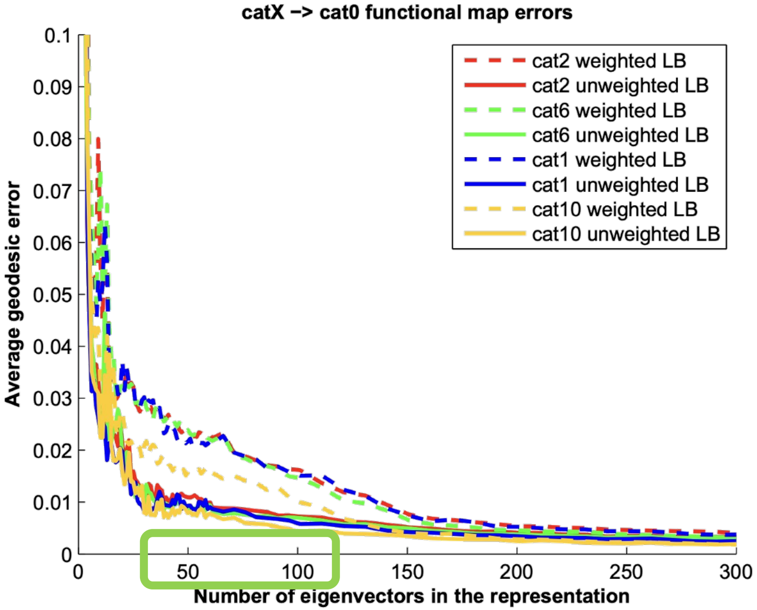
$$\mathrm{P} = \Phi_1 \cdot C \cdot \Phi_2^{\dagger}$$

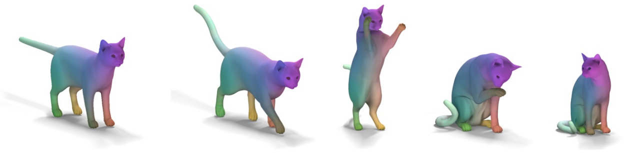
A functional map is a rank-k approximation of a point map

Accuracy of the Functional Map

Average mapping error vs. number of basis used

- In practice, somewhere between 20 to 100 basis are sufficient





(a) *target* **(b)** *Cat10* **(c)** *Cat1* **(d)** *Cat2* **(e)** *Cat6*

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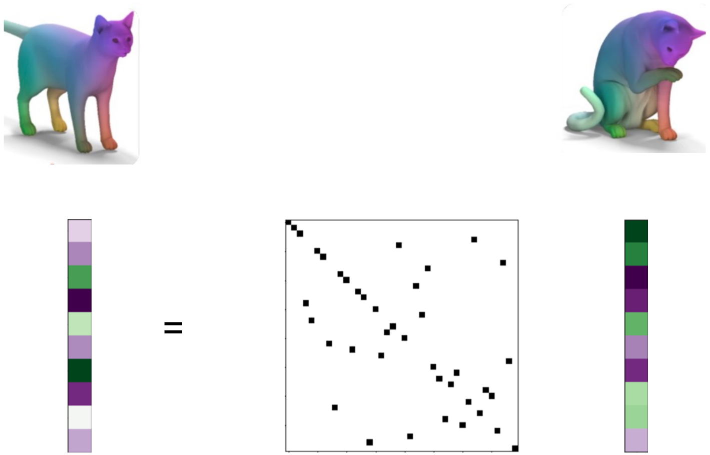
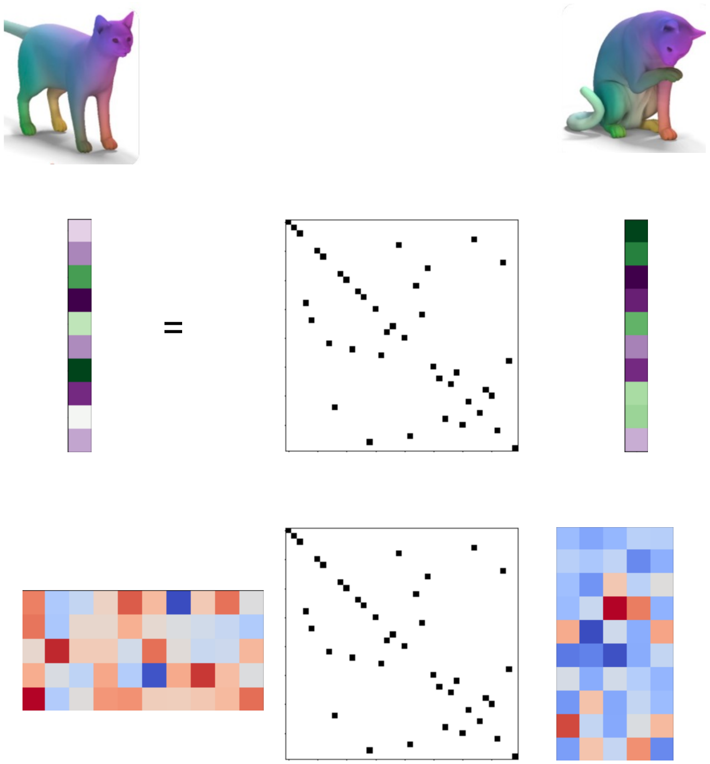
Focus on the **input** and **output** of the matrix



A point map transfer functions between two shapes

Focus on the **input** and **output** of the matrix



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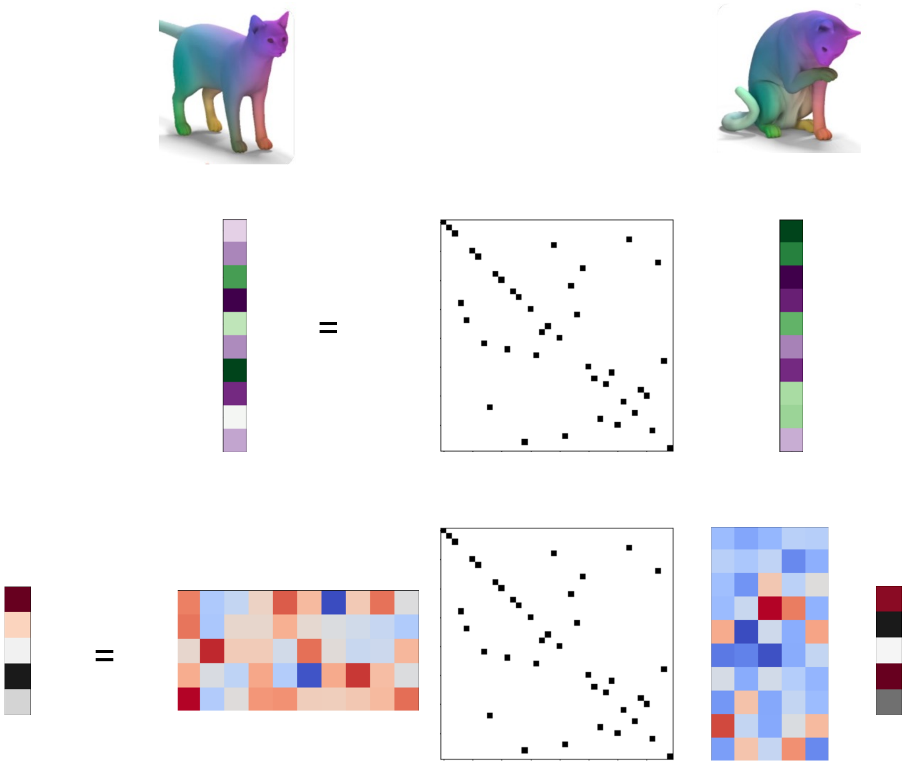

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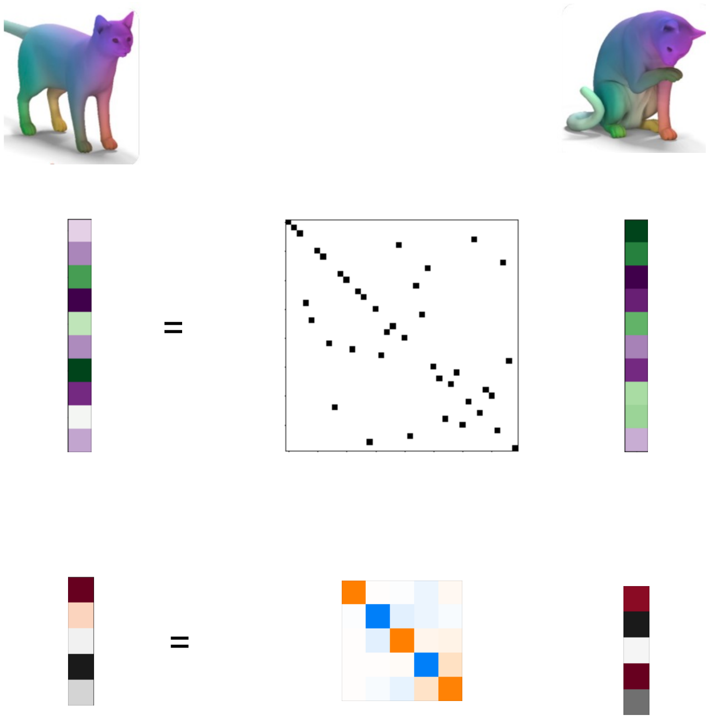
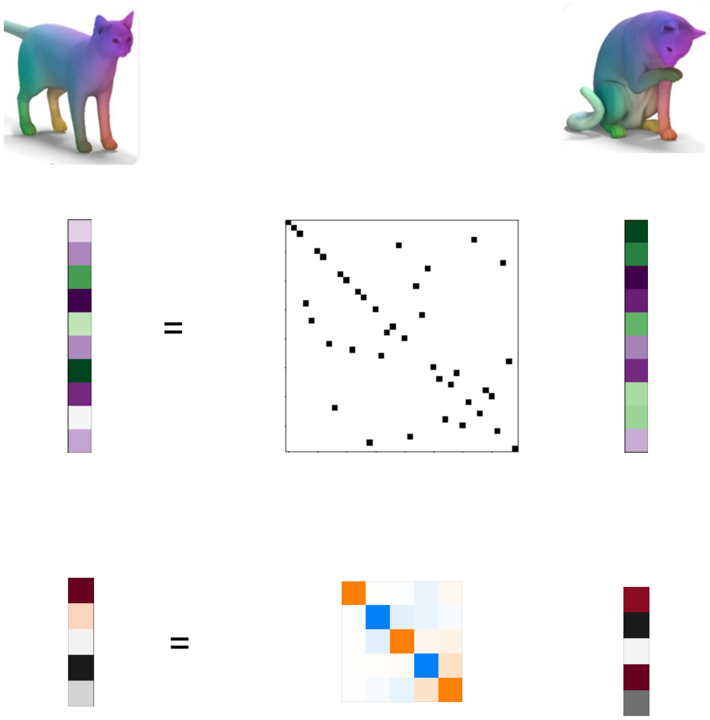
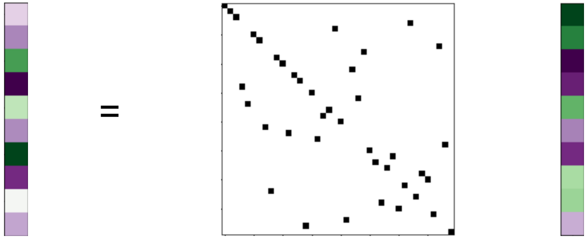


Focus on the **input** and **output** of the matrix



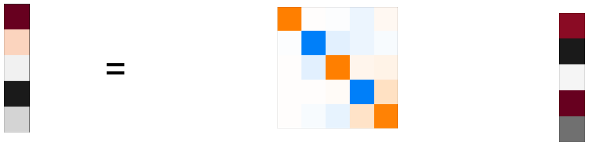
A functional map translates coefficients of functions between two shapes

Focus on the **input** and **output** of the matrix



Spatial domain

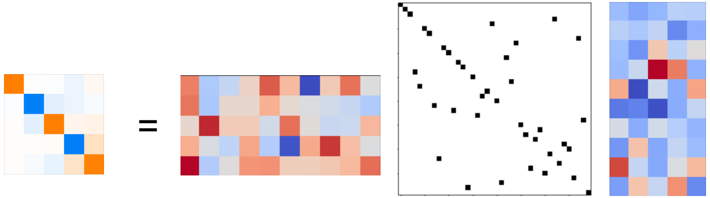
A point map transfer functions between two shapes



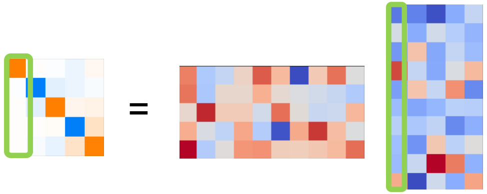
Spectral domain

A functional map translates coefficients of functions between two shapes

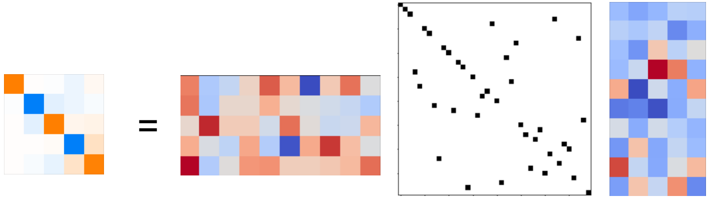
$$\mathrm{b} = \mathrm{C} \cdot a$$



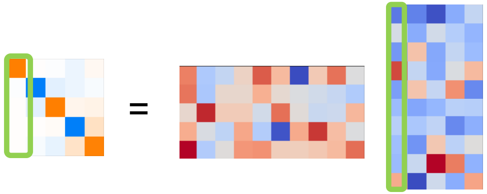
$$C = \Phi_1^{\dagger} \cdot P \cdot \Phi_2$$



$$C = \Phi_1^{\dagger} \cdot \Phi_{2a}$$



$$C = \Phi_1^{\dagger} \cdot P \cdot \Phi_2$$

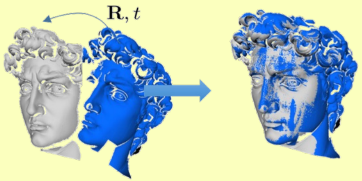


$$C = \Phi_1^{\dagger} \cdot \Phi_{2a}$$

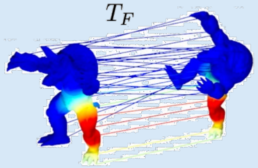
$$\Phi_1 \cdot C = \Phi_{2a}$$

Spectral Rigid Alignment

Rigid



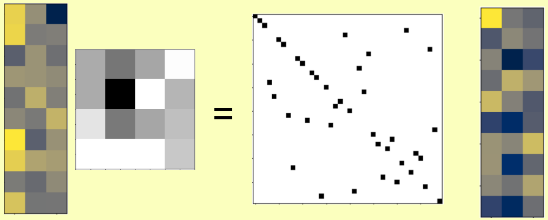
Non-rigid



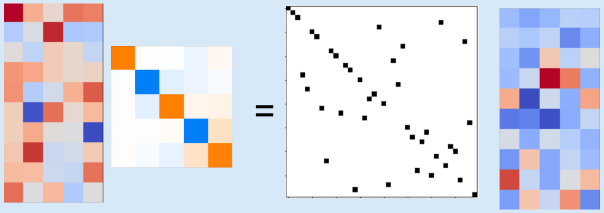
4x4 Rt

kxk C

aligns
xyz
coordinates



aligns
spectral
embeddings



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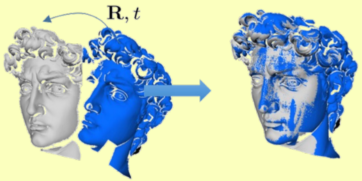
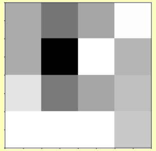
Spectral Rigid Alignment

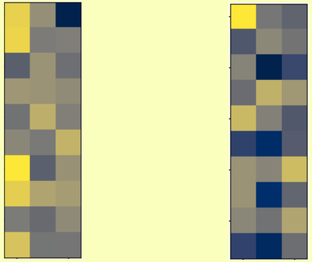
Rigid



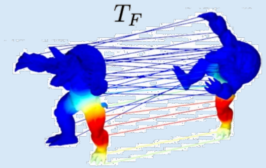
4x4 Rt



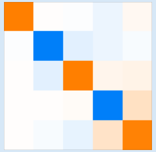
aligns
xyz
coordinates



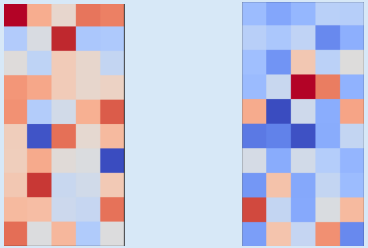
Non-rigid



kxk C



aligns
spectral
embeddings



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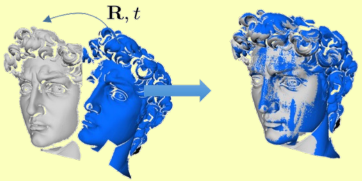
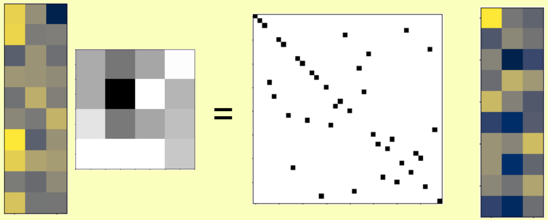
Spectral Rigid Alignment

Rigid



4x4 Rt

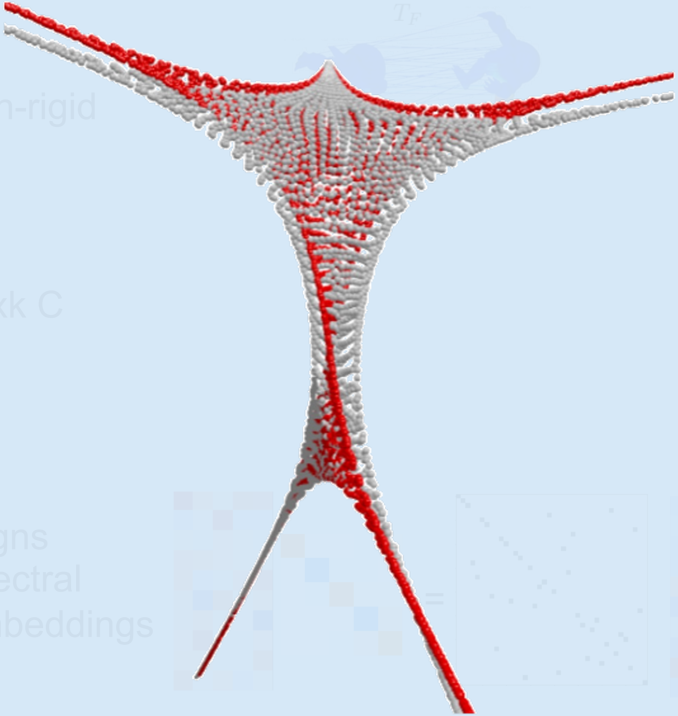
aligns
xyz
coordinates



Non-rigid

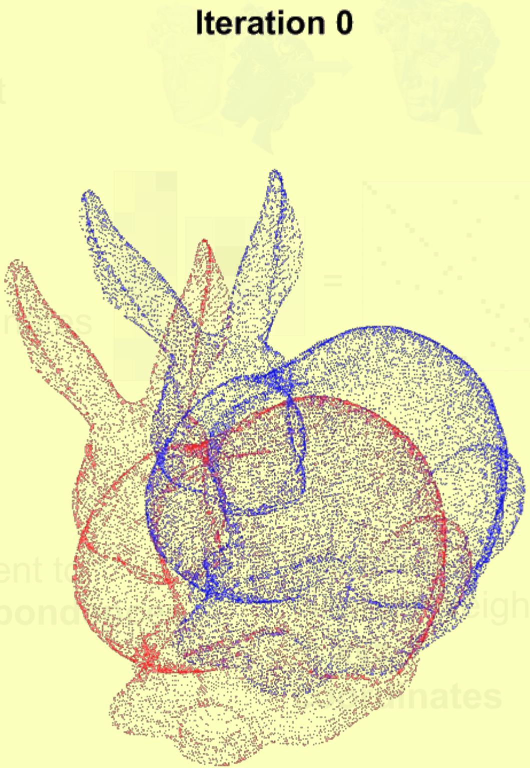
kxk C

aligns
spectral
embeddings



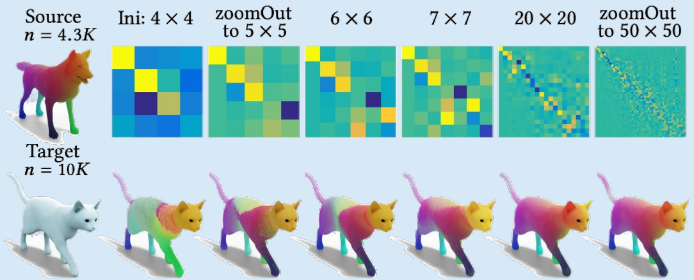
Ovsjanikov, M., Corman, E., Bronstein, M., Rodolà, E., Ben-Chen, M., Guibas, L., ... & Bronstein, A. (2016). Computing and processing correspondences with functional maps. In *SIGGRAPH ASIA 2016 Courses* (pp. 1-60).

Iterative Closest Point



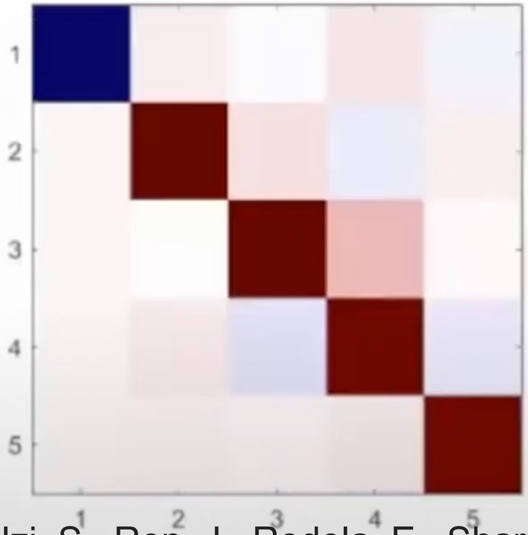
https://www.youtube.com/watch?v=uzOCS_gdZuM

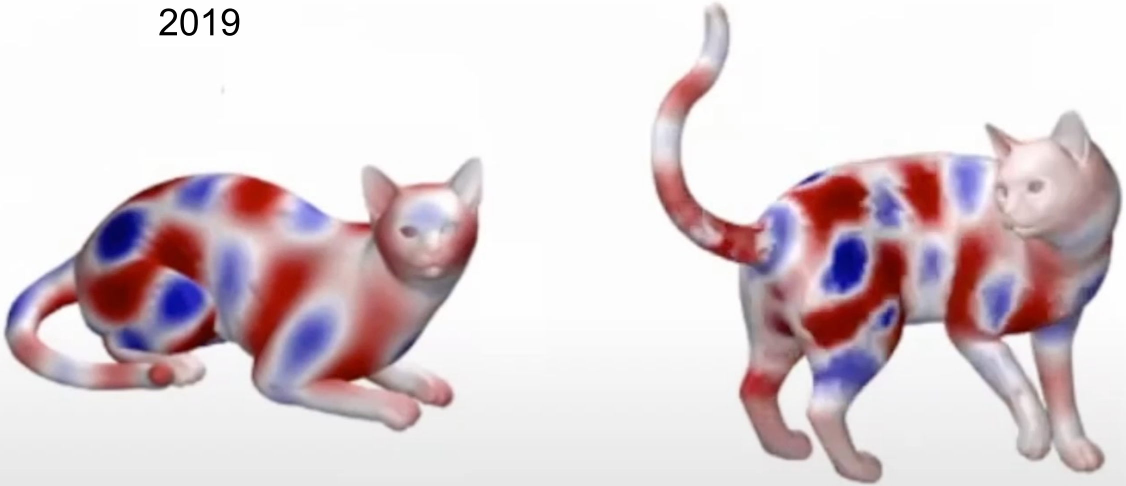
Map Refinement: ZoomOut



Map Refinement

2019



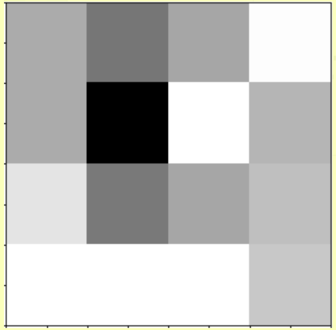


Melzi, S., Ren, J., Rodola, E., Sharma, A., Wonka, P., & Ovsjanikov, M. (2019). Zoomout: Spectral upsampling for efficient shape correspondence. *arXiv preprint arXiv:1904.07865*.

Spectral Rigid Alignment

Rigid
4x4 Rt

aligns xyz coordinates

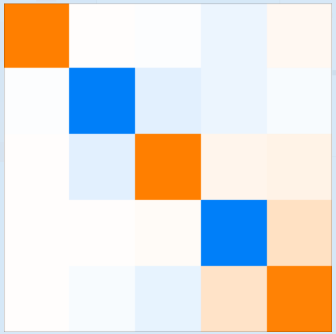


Alignment to **correspondences**

Search for Nearest Neighbor in **xyz coordinates**

Non-rigid
kxk C

aligns spectral embeddings

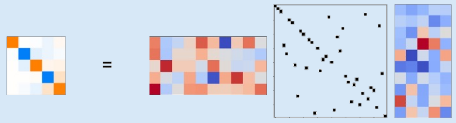


Alignment to **correspondences**

Search for Nearest Neighbor In **spectral embeddings**

Ovsjanikov, M., Corman, E., Bronstein, M., Rodolà, E., Ben-Chen, M., Guibas, L., ... & Bronstein, A. (2016). Computing and processing correspondences with functional maps. In *SIGGRAPH ASIA 2016 Courses* (pp. 1-60).

Functional Maps



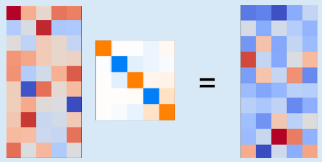
$$C = \Phi_1^{\dagger} \cdot P \cdot \Phi_2$$

Approximation
of Point Map

linear, compact
and flexible

$$b = C \cdot a$$

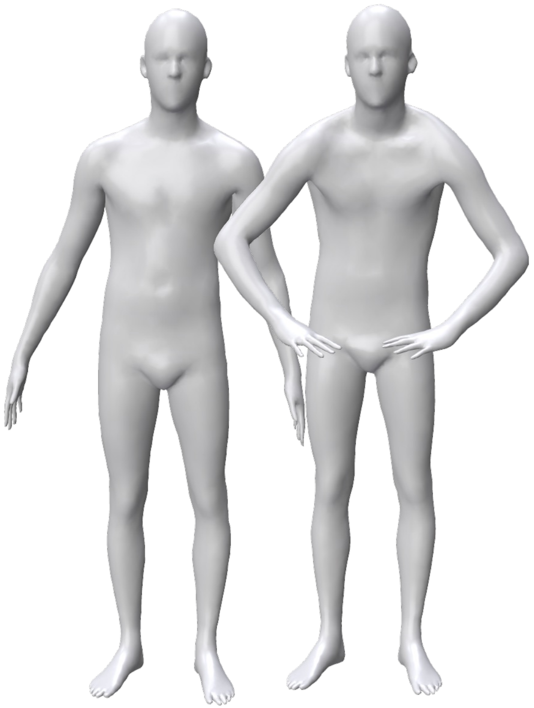
Translates coefficients



$$\Phi_1 \cdot C = \Phi_{2a}$$

Aligns Bases

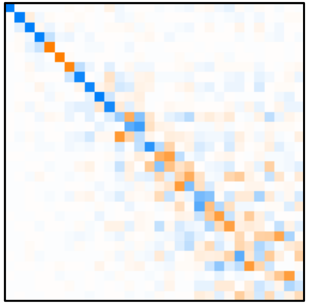
What is the magic?



isometric

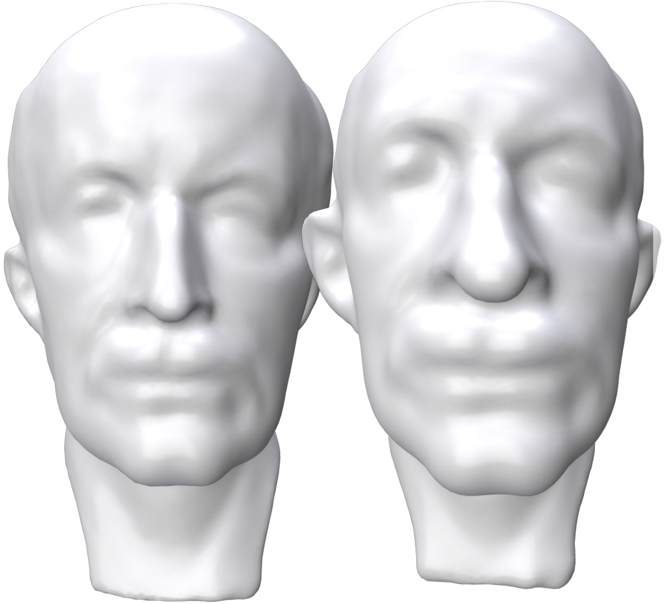
Eigenfunctions of Laplace-Beltrami Operator

Invariance under non-rigid isometric deformations



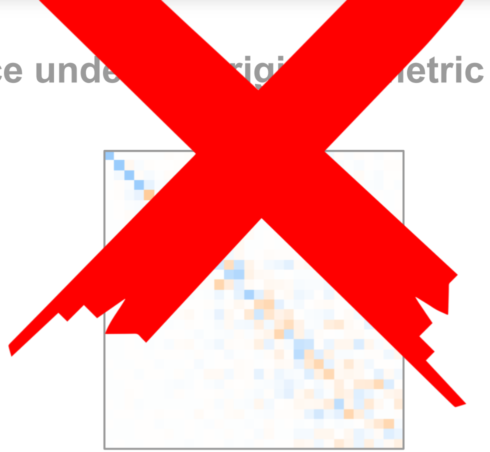
Basis functions exhibit similar patterns, which can be matched

What is the magic?



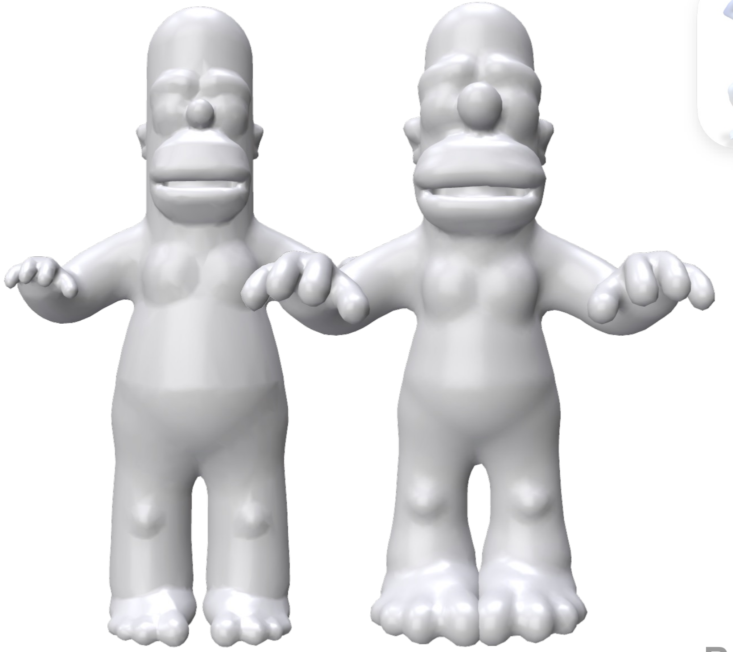
Non-isometric

Invariance unde ... rigi ... etric deformations



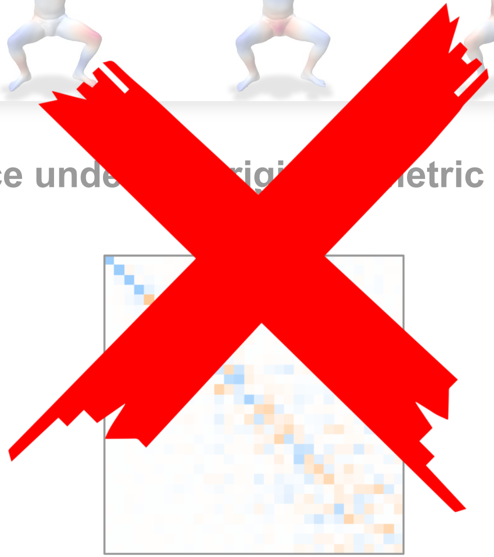
Basis functions exhibit similar patterns, which can be matched

What is the magic?



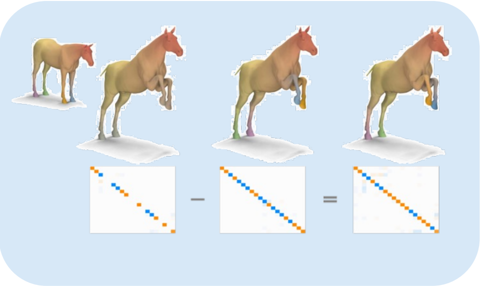
Non-isometric

Invariance unde rigi etric deformations

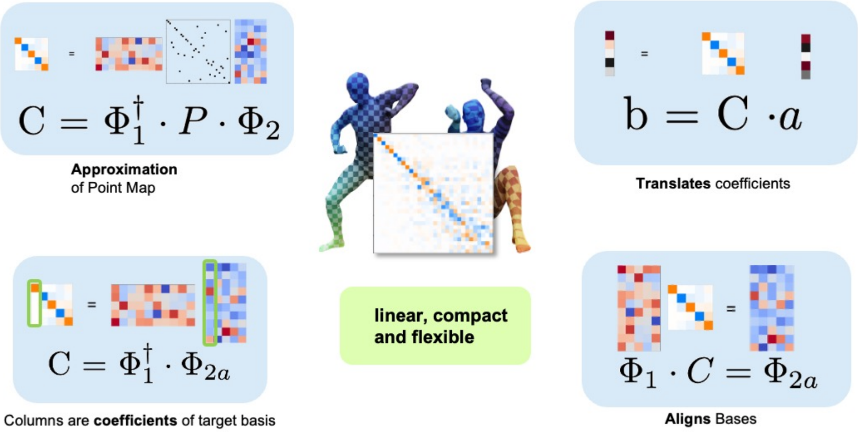


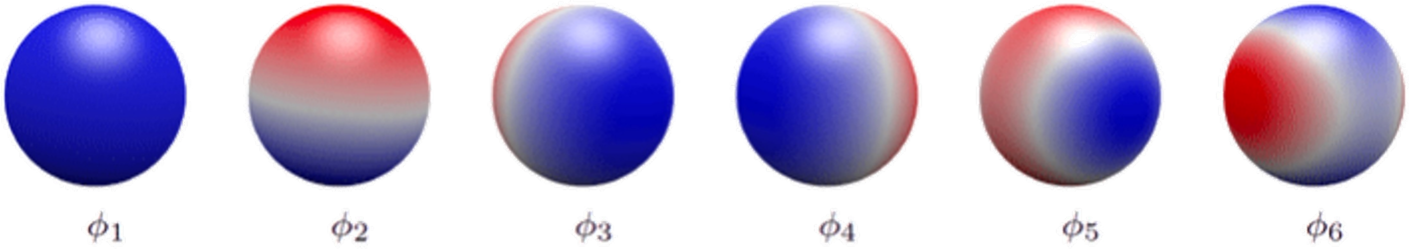
Basis functions exhibit similar patterns, which can be matched

Background Work



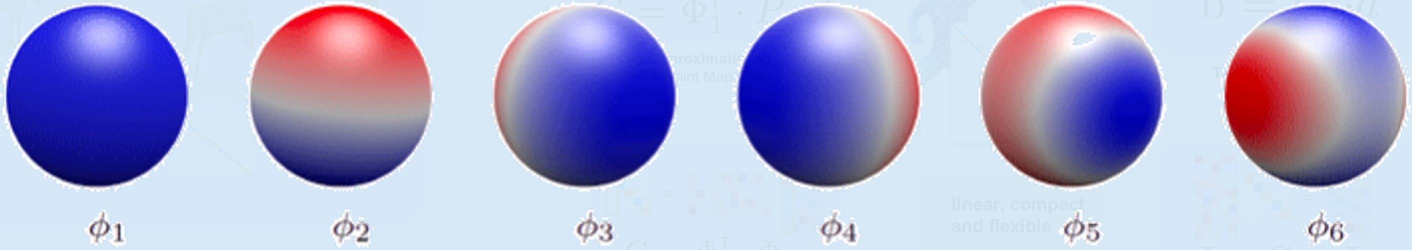
Functional Maps [Ovsjanikov et al. 2012]



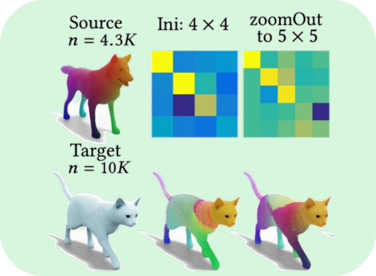


Melzi, Simone, Jing Ren, Emanuele Rodola, Abhishek Sharma, Peter Wonka, and Maks Ovsjanikov. "Zoomout: Spectral upsampling for efficient shape correspondence." *arXiv preprint arXiv:1904.07865* (2019).

Background Work



axiomatic



ZoomOut [Melzi et al. 2019]

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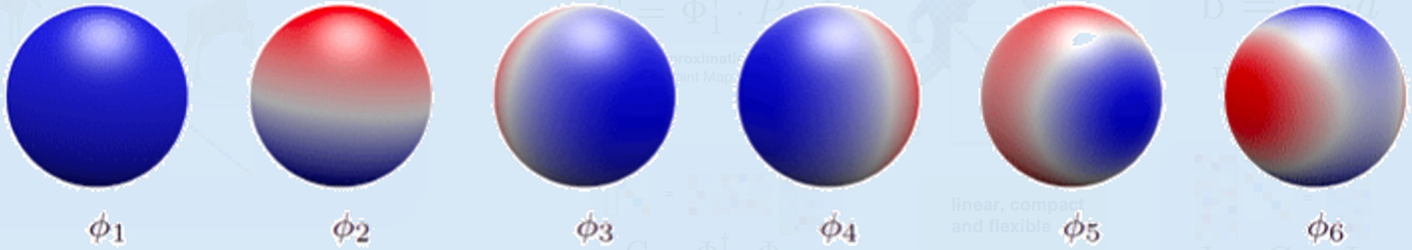
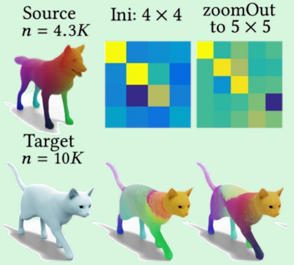
Background Work



axiomatic



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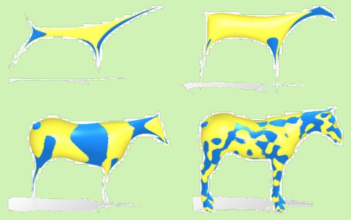
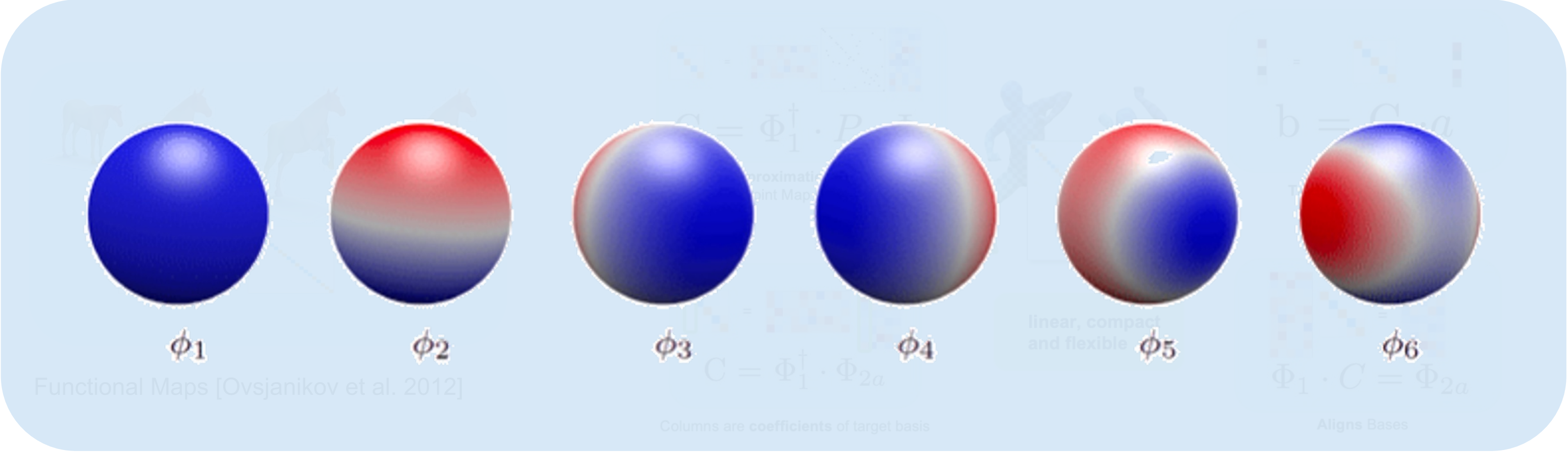


Smooth Shells [Eisenberger et al. 2020]

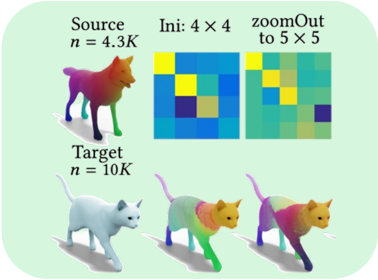
Eisenberger, Marvin, Zorah Lahner, and Daniel Cremers. "Smooth shells: Multi-scale shape registration with functional maps." *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*. 2020.

Background Work

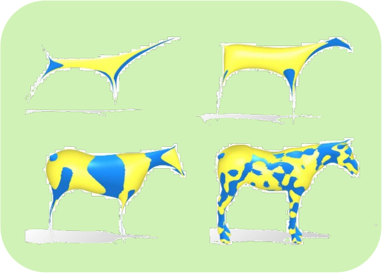


axiomatic

supervised



ZoomOut [Melzi et al. 2019]

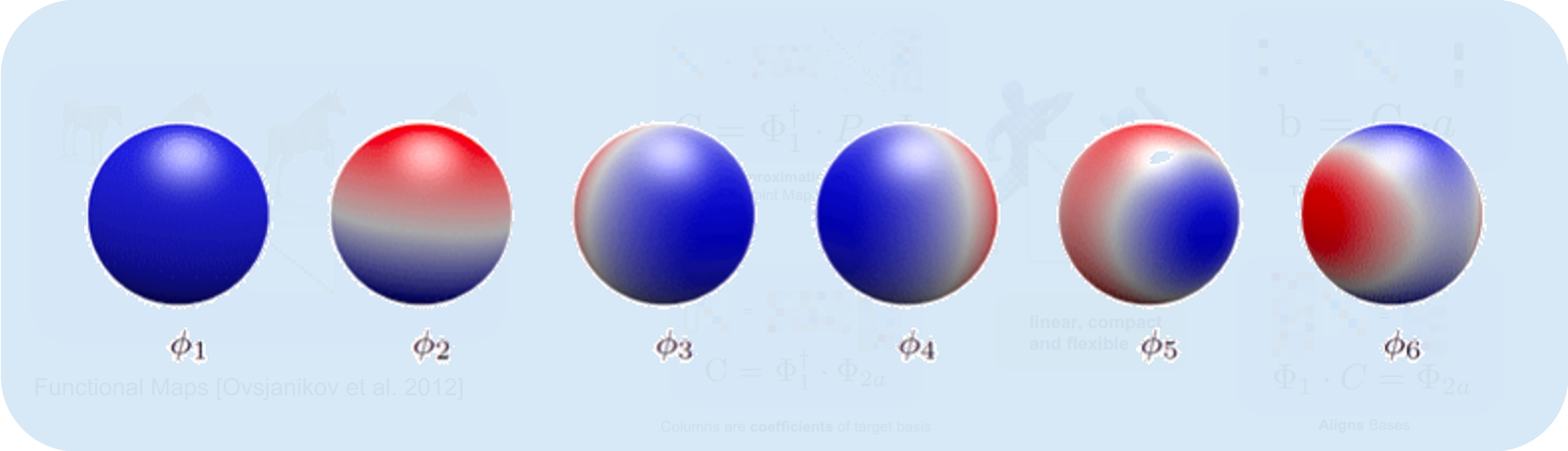


Smooth Shells [Eisenberger et al. 2020]

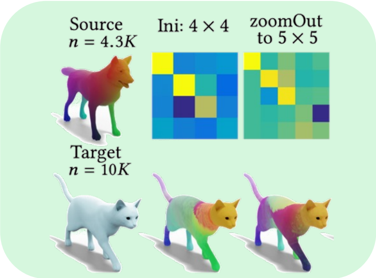
GeomFmaps [Donati et al. 2020]

Donati, Nicolas, Abhishek Sharma, and Maks Ovsjanikov. "Deep geometric functional maps: Robust feature learning for shape correspondence." *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, 2020.

Background Work



axiomatic | **supervised** | **unsupervised**



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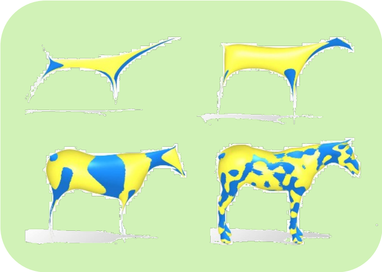

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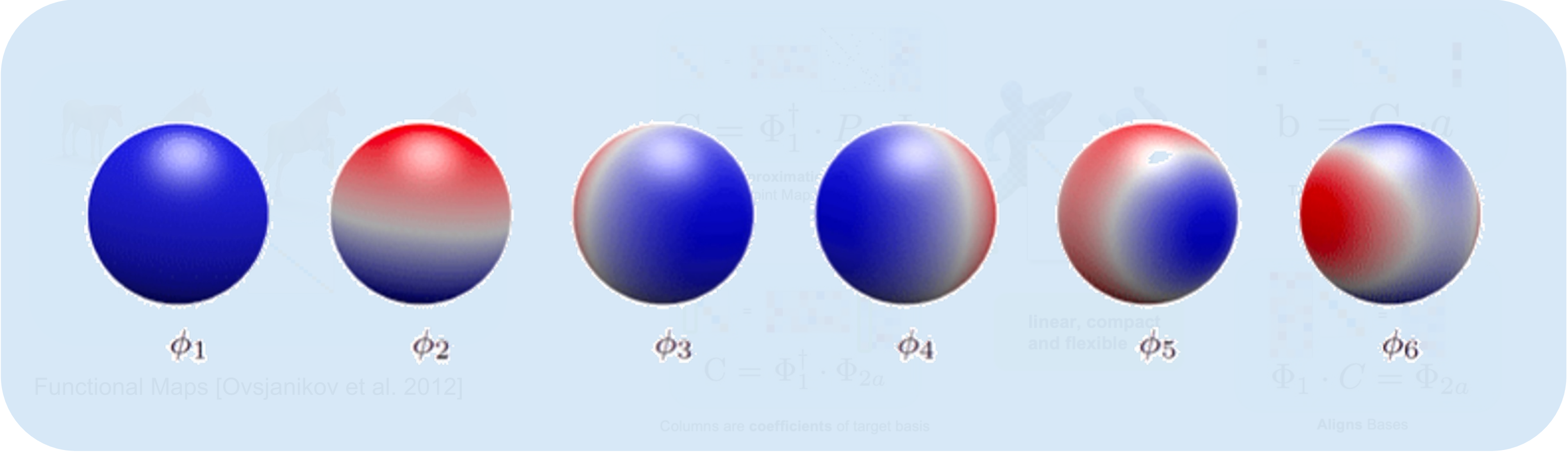
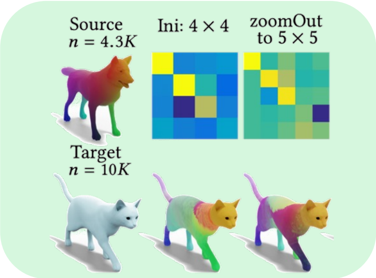
ULRSSM [Cao et al. 2023]

Cao, D., Roetzer, P., & Bernard, F. (2023). Unsupervised learning of robust spectral shape matching. ACM Transactions on Graphics (TOG).
https://doi.org/10.1145/3592107

Background Work



axiomatic | **supervised** | **unsupervised**



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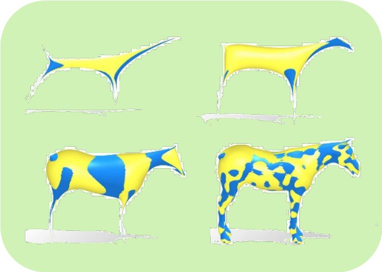

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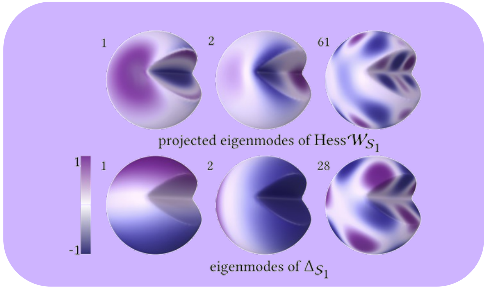
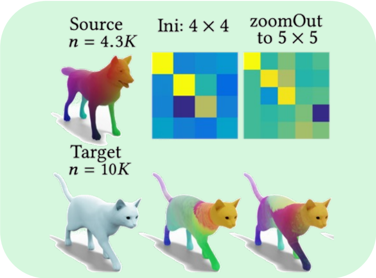
Background Work

- Crease Awareness
- Non-orthogonal Basis
- Generalized FMap Framwork

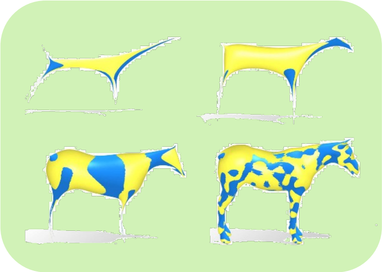


An Elastic Basis [Hartwig et al. 2023]

axiomatic | **supervised** | **unsupervised**



ZoomOut [Melzi et al. 2019]



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Background Work

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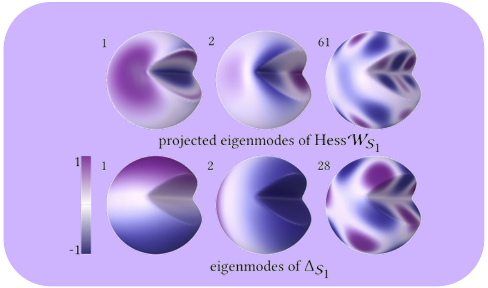

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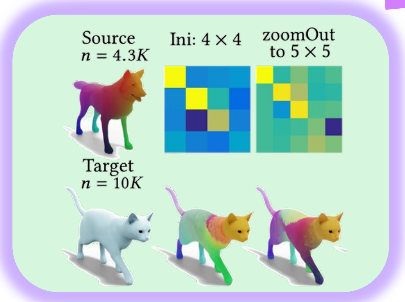

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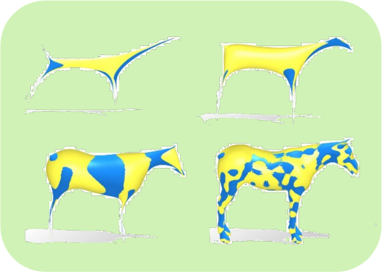

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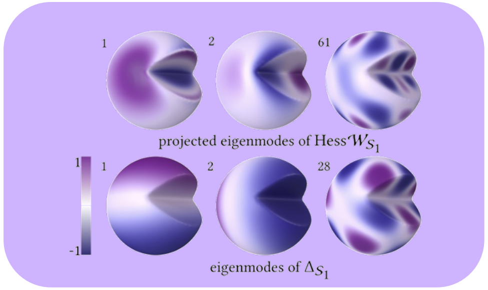
An Elastic Basis

Elastic Energy

membrane contribution (intrinsic) — bending contribution (extrinsic)

$$\mathcal{W}_{\mathcal{S}}[\psi] = \mathcal{W}_{\text{mem}}[\psi] + \mathcal{W}_{\text{bend}}[\psi],$$

for a deformation $\psi \in (\mathcal{F}(\mathcal{S}))^3$



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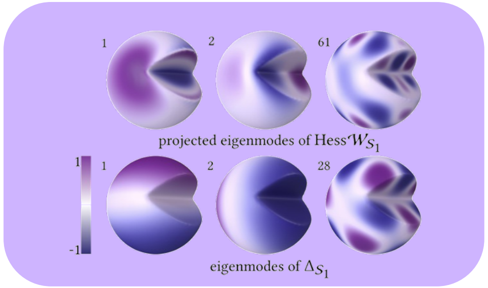
An Elastic Basis

$$\operatorname{Hess} \mathcal{W}_{\mathcal{S}}[\mathrm{Id}]$$



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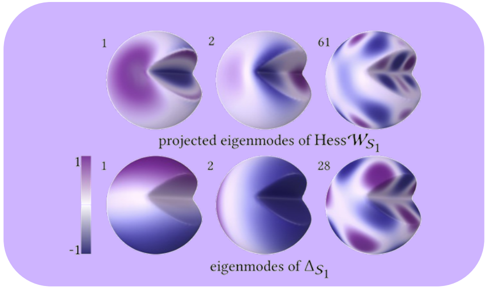
An Elastic Basis

solutions of the eigenfunction problem

$$\mathrm{Hess}\,\mathcal{W}_{\mathcal{S}}[\mathrm{Id}]\psi_i = \lambda_i \psi_i$$

[Hildebrandt et al. 2010]



An Elastic Basis [Hartwig et al. 2023]

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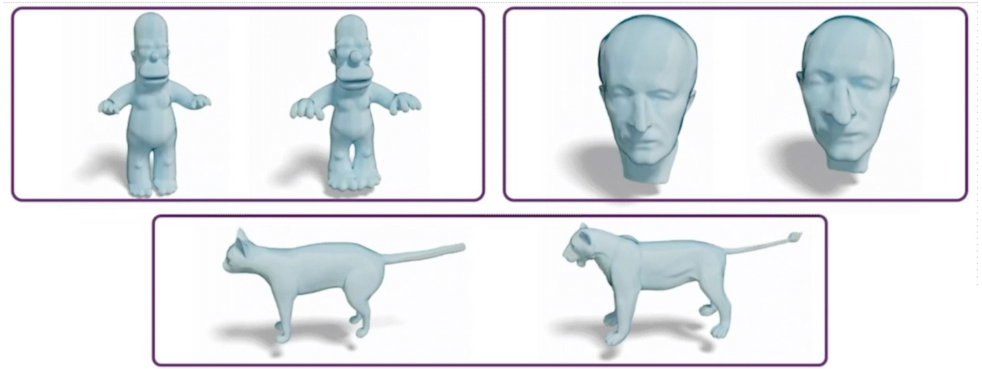
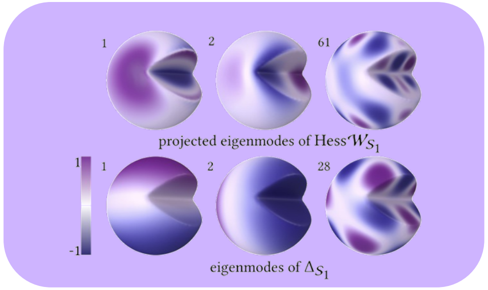
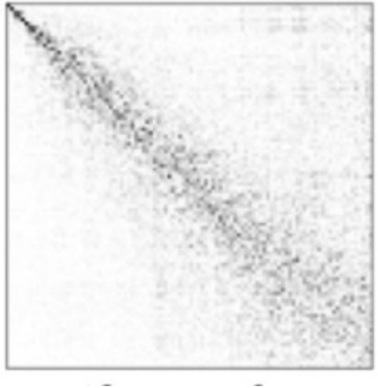




An Elastic Basis [Hartwig et al. 2023]



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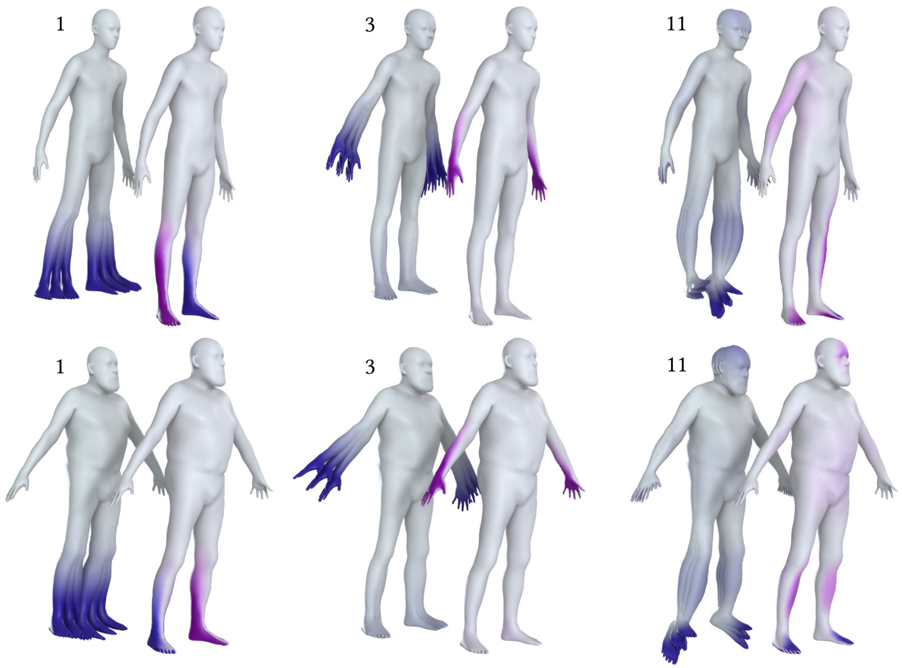

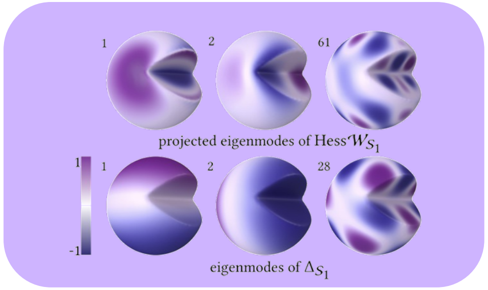

An Elastic Basis [Hartwig et al. 2023]

projection on vertex normals

$$\phi_i \in \mathcal{F}(\mathcal{S})$$

$\phi_1, \phi_2, \ldots,$ not orthogonal

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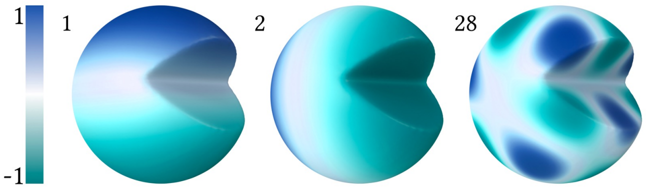
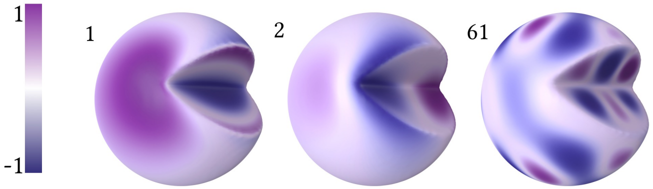
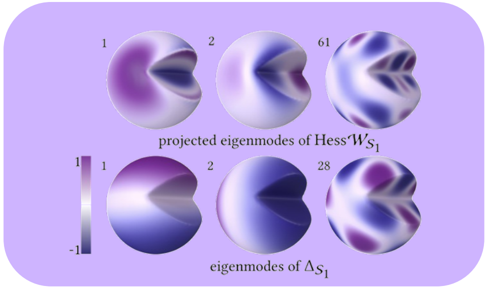
An Elastic Basis



Classical LB Basis: purely intrinsic



Elastic Basis: extrinsic aware



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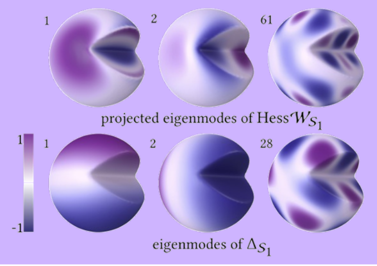
An Elastic Basis

- Non-orthogonal Basis
- Generalized FMap Framework



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Generalized FMap Framework

$$\Phi_k^T M \Phi_k = I$$

Mass matrix w.r.t. the reduced basis

$$\Phi_k^T M \Phi_k = M_k$$

Hartwig, F., Sassen, J., Azencot, O., Rumpf, M., & Ben-Chen, M. (2023, July). An Elastic Basis for Spectral Shape Correspondence. In *ACM SIGGRAPH 2023 Conference Proceedings* (pp. 1-11).

Generalized FMap Framework

$$\Phi_k^T M \Phi_k = I$$

Mass matrix w.r.t. the reduced basis

$$\Phi_k^T M \Phi_k = M_k$$

$$f = \Phi_k x \qquad g = \Phi_k y$$

$$\begin{aligned} \langle f, g \rangle_M &= f^T M g \\ &= (\Phi_k x)^T M (\Phi_k y) \\ &= x^T y \end{aligned}$$

Scalar dot product

$$f = \Phi_k x \qquad g = \Phi_k y$$

$$\begin{aligned} \langle f, g \rangle_M &= f^T M g \\ &= (\Phi_k x)^T M (\Phi_k y) \\ &= x^T M_k y \end{aligned}$$

Hartwig, F., Sassen, J., Azencot, O., Rumpf, M., & Ben-Chen, M. (2023, July). An Elastic Basis for Spectral Shape Correspondence. In *ACM SIGGRAPH 2023 Conference Proceedings* (pp. 1-11).

Generalized FMap Framework

Standard		Generalized
$\Phi_k^\dagger = (\Phi_k^T M \Phi_k)^{-1} \Phi_k^T M = \Phi_k M$	Pseudo-inverse	$\Phi_k^\dagger = (\Phi_k^T M \Phi_k)^{-1} \Phi_k^T M = M_k^{-1} \Phi_k M$
$C_{12} = \Phi_1^\dagger P_{12} \Phi_2$	Functional Map	$C_{12} = \Phi_1^\dagger P_{12} \Phi_2$
$\langle x, C_{12} y \rangle = \langle C_{12}^* x, y \rangle$ $C_{12}^* = C_{12}^T$	Adjoint	$\langle x, C_{12} y \rangle_{M_{1,k}} = \langle C_{12}^* x, y \rangle_{M_{2,k}}$ $C_{12}^* = M_{2,k}^{-1} C_{12}^T M_{1,k}$

Hartwig, F., Sassen, J., Azencot, O., Rumpf, M., & Ben-Chen, M. (2023, July). An Elastic Basis for Spectral Shape Correspondence. In *ACM SIGGRAPH 2023 Conference Proceedings* (pp. 1-11).

Generalized FMap Framework

$$\|C_{12}\|_F^2 = \mathrm{tr}(C_{12}^T C_{12})$$

Frobenius Norm

Operator Norm

$$\|C_{12}\|_{HS}^2 = \mathrm{tr}(C_{12}^* C_{12})$$
$$= \|M_{1,k}^{\frac{1}{2}} C_{12} M_{2,k}^{-\frac{1}{2}}\|_F^2$$

Hilbert-Schmidt Norm

Hartwig, F., Sassen, J., Azencot, O., Rumpf, M., & Ben-Chen, M. (2023, July). An Elastic Basis for Spectral Shape Correspondence. In *ACM SIGGRAPH 2023 Conference Proceedings* (pp. 1-11).

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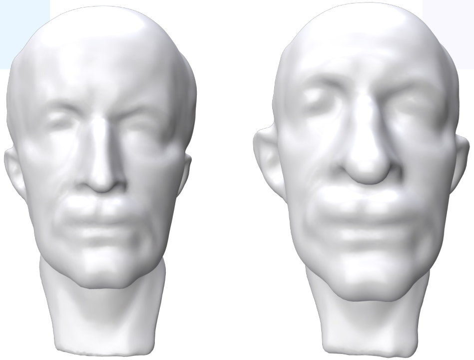
Frobenius Norm



Hilbert-Schmidt Norm

$$\|C_{12}\|_{M_{2,k}}^2 = tr(M_{2,k})$$

$$\|C_{12}\|_{HS}^2 = tr(M_{2,k}^{-1} M_{1,k})$$

Hartwig, F., Sassen, J., Azencot, O., Rumpf, M., & Ben-Chen, M. (2023, July). An Elastic Basis for Spectral Shape Correspondence. In *ACM SIGGRAPH 2023 Conference Proceedings* (pp. 1-11).

Generalized FMap Framework adapted to ZoomOut

ZoomOut Objective

$$\|C_{12}C_{12}^{T} - I\|_{F}$$

$$\|C_{12}C_{12}^{*} - I\|_{HS}$$

ZoomOut Algorithm

while spectrally upsampling

1. **Correspondence**

$\Phi_1 \qquad \Phi_2 C_{12}^{T}$

Via Nearest Neighbor Search

2. **Functional Map**

$$C_{12} = \Phi_1^{\dagger} P_{12} \Phi_2$$

while spectrally upsampling

1. **Correspondence**

$\Phi_1 M_{1,k}^{-\frac{1}{2}} \qquad \Phi_2 C_{12}^{*} M_{1,k}^{-\frac{1}{2}}$

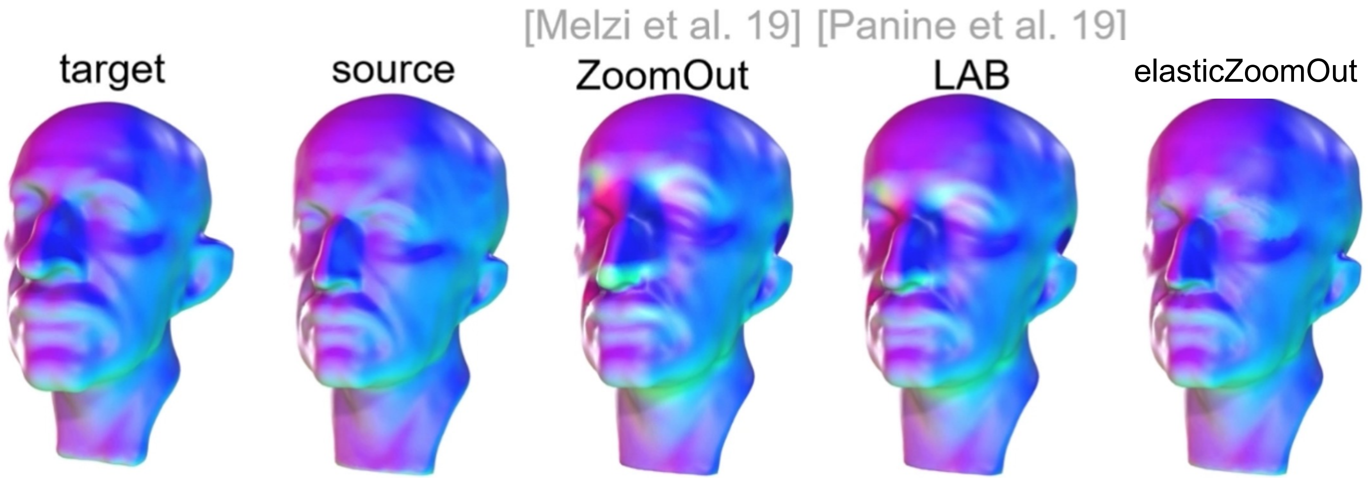
Via Nearest Neighbor Search

2. **Functional Map**

$$C_{12} = \Phi_1^{\dagger} P_{12} \Phi_2$$

Hartwig, F., Sassen, J., Azencot, O., Rumpf, M., & Ben-Chen, M. (2023, July). An Elastic Basis for Spectral Shape Correspondence. In *ACM SIGGRAPH 2023 Conference Proceedings* (pp. 1-11).

Generalized FMap Framework adapted to ZoomOut



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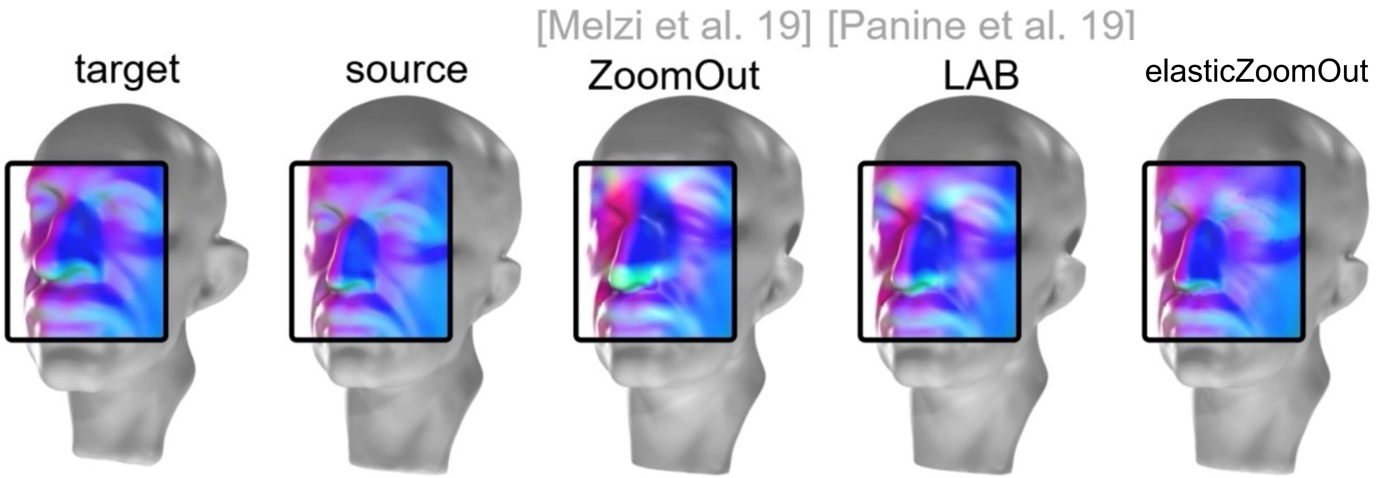

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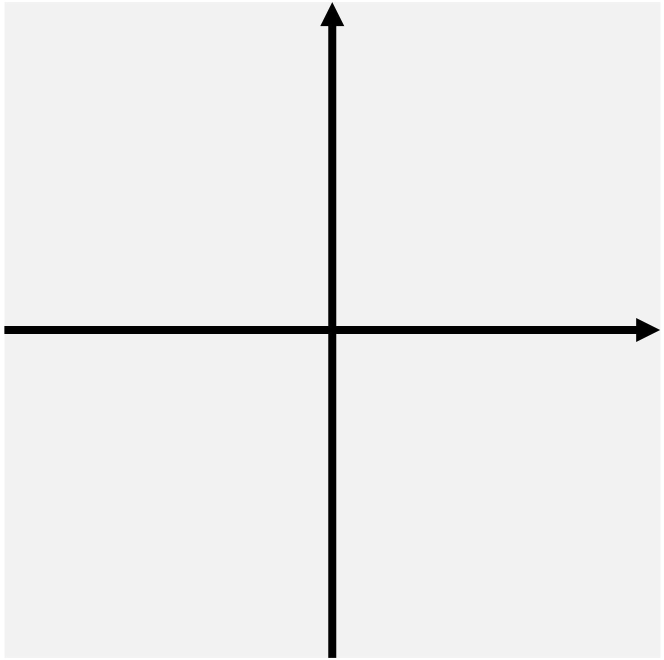
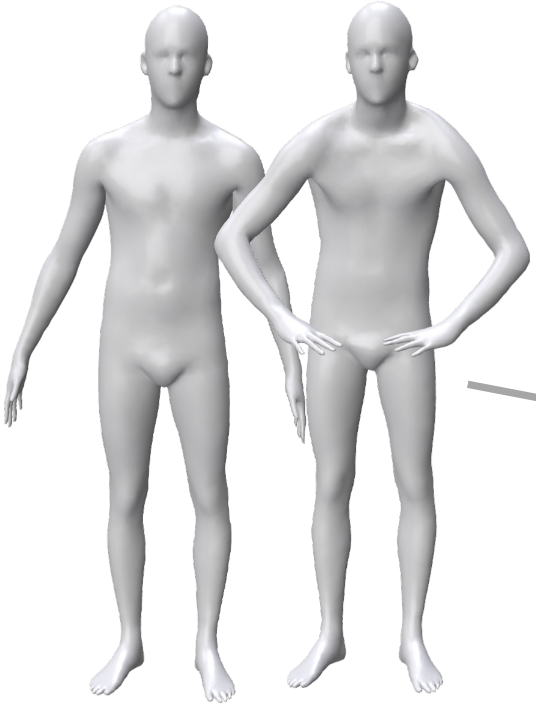
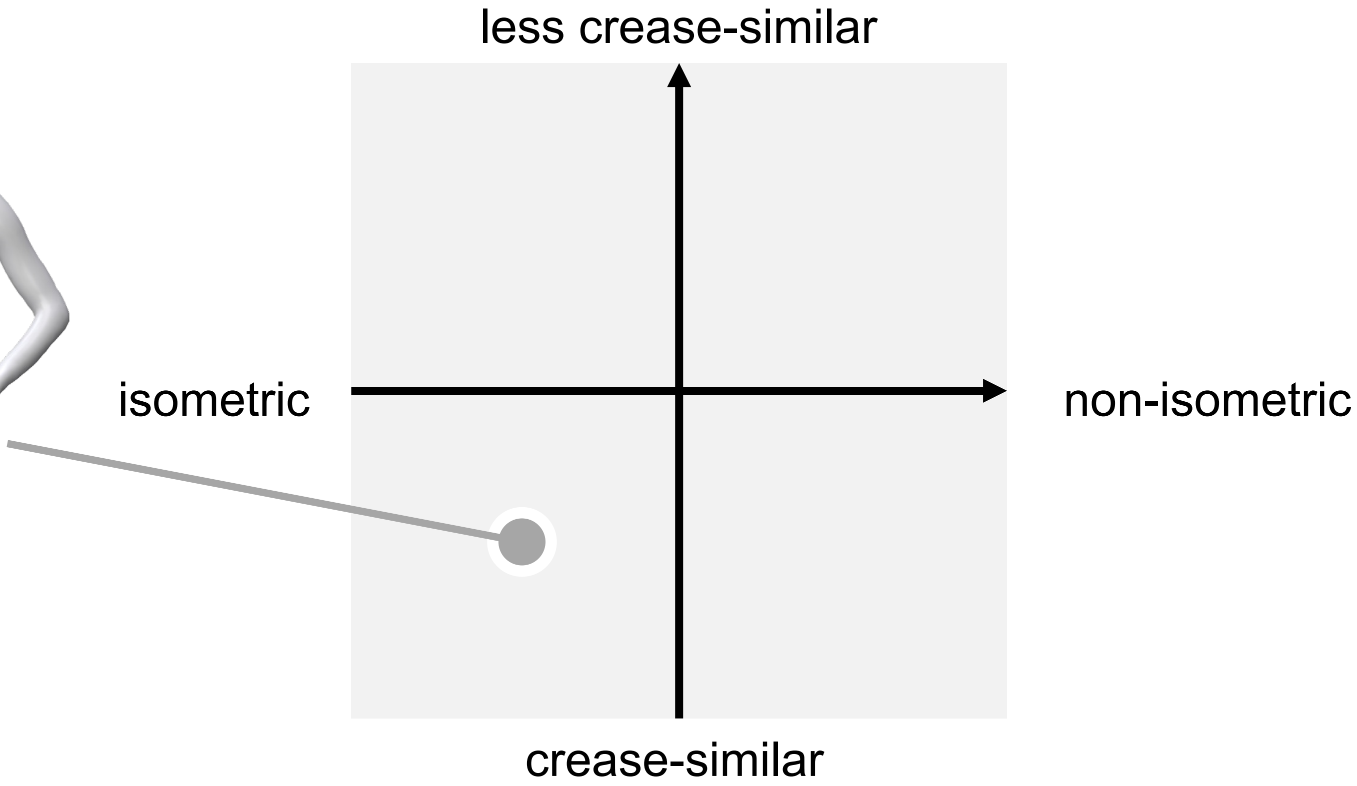
Challenge


less crease-similar
isometric
non-isometric
crease-similar

Challenge

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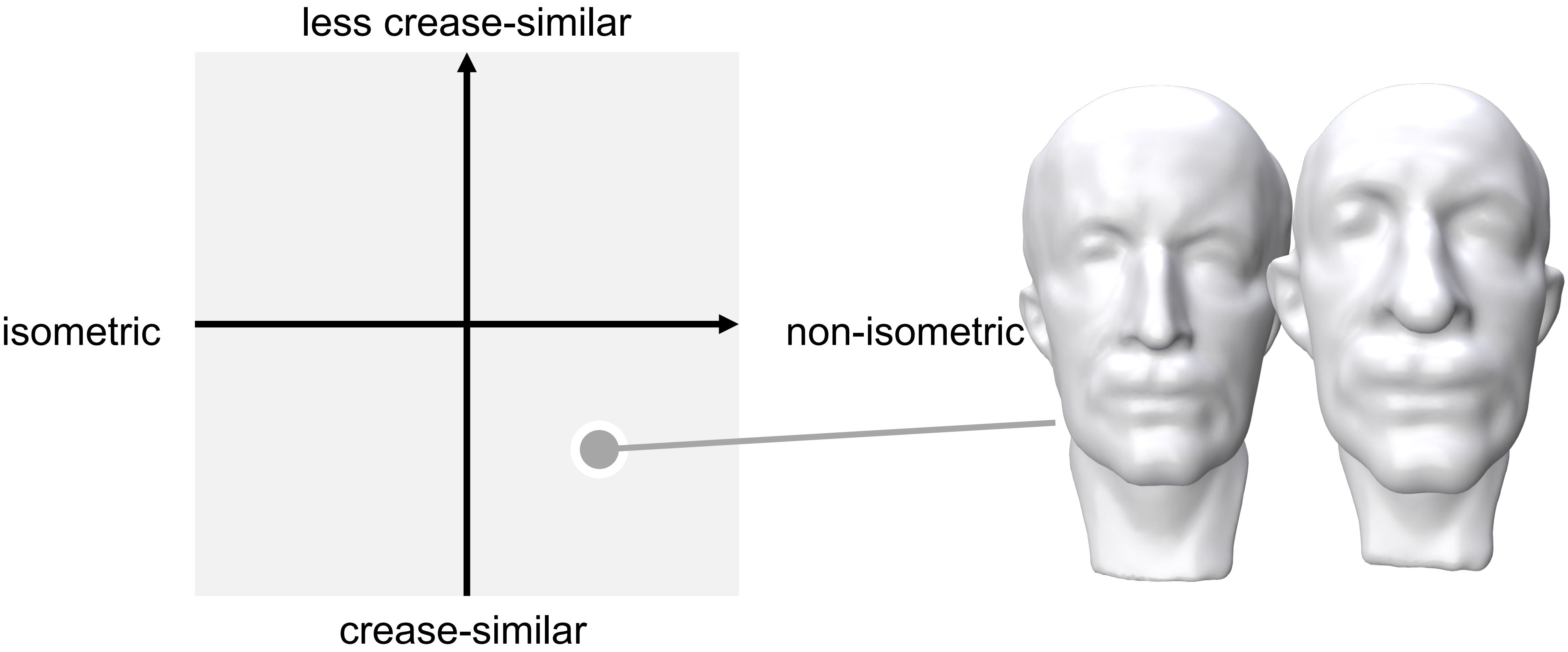
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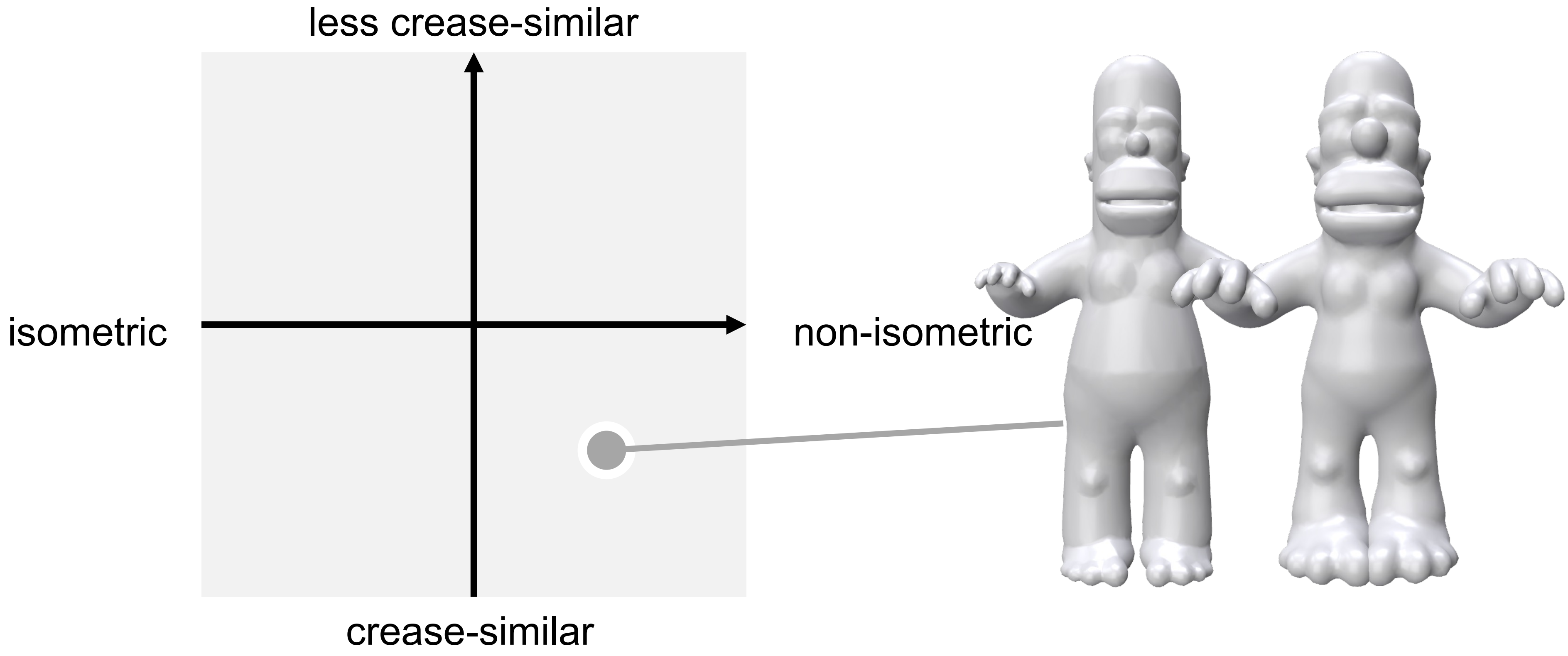

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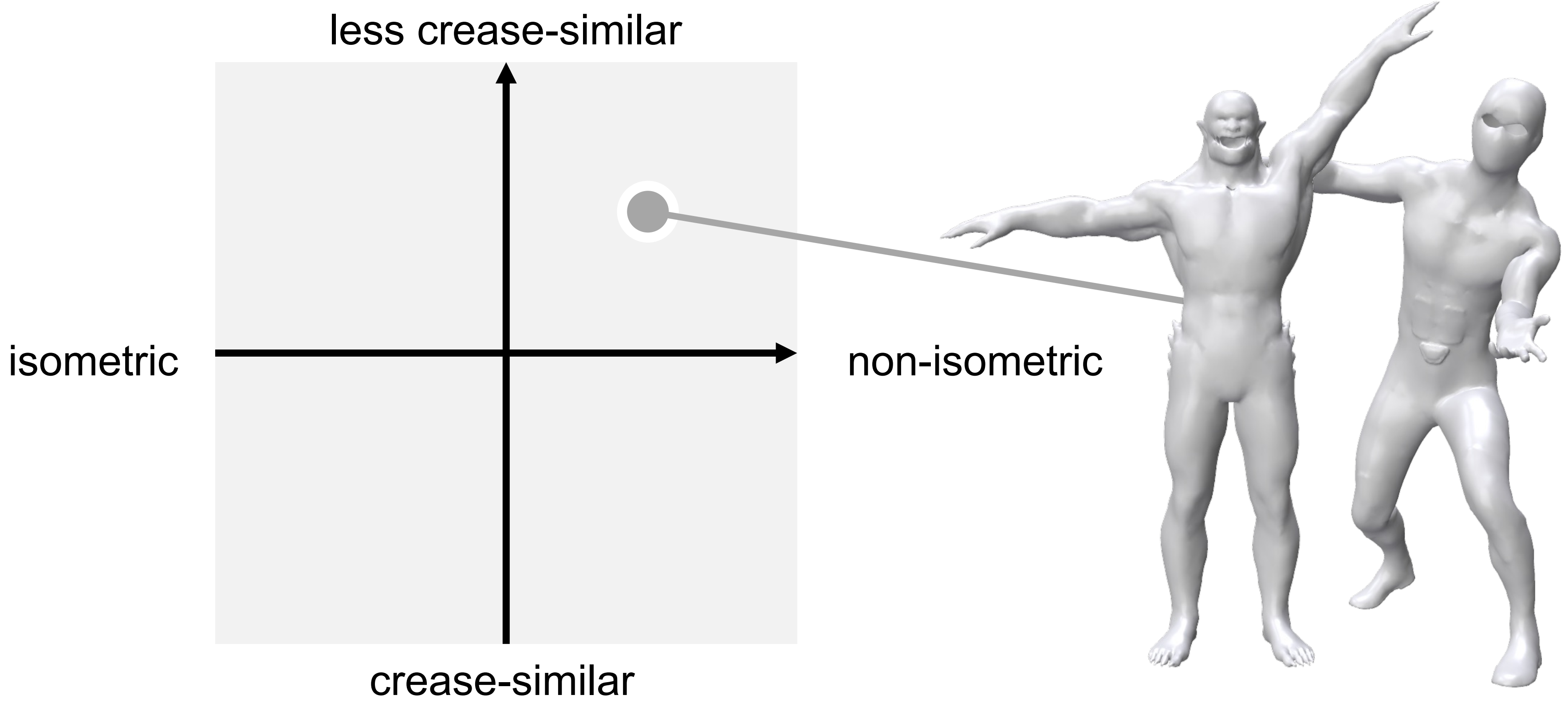

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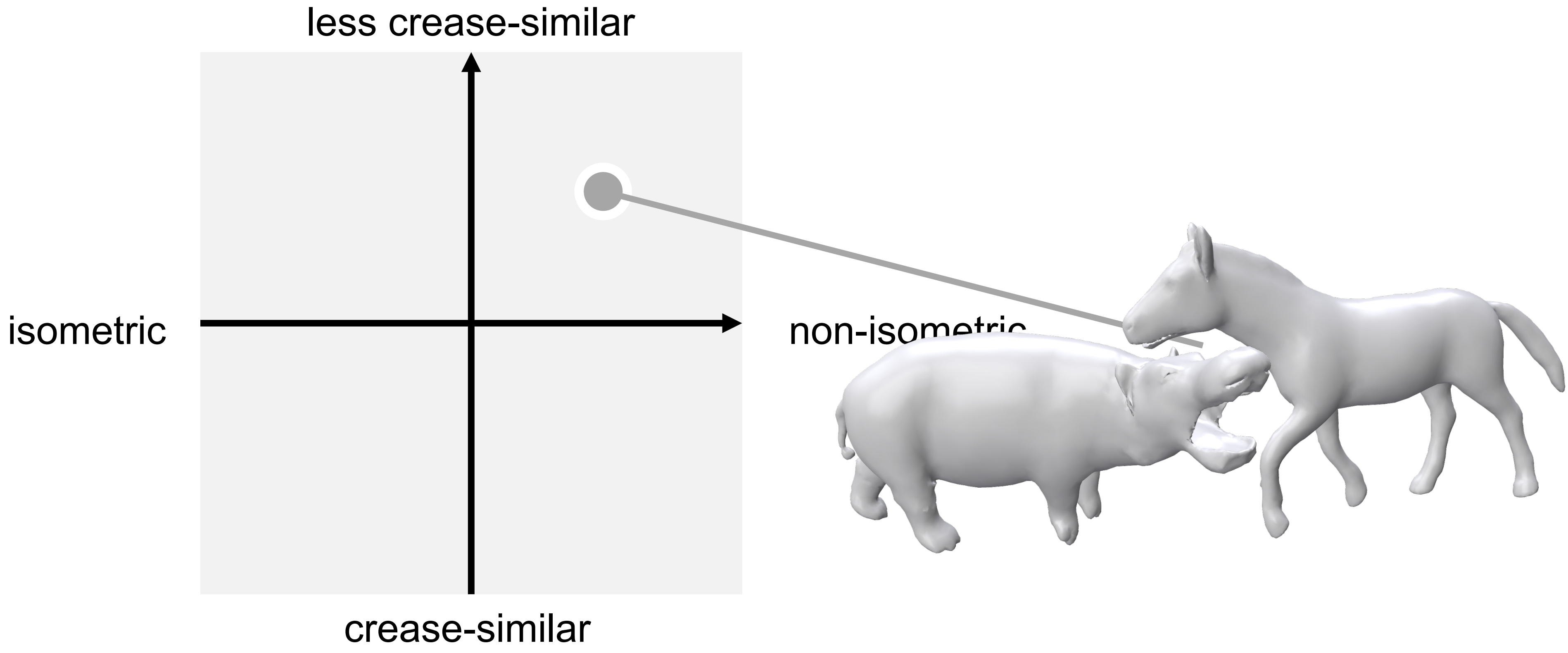

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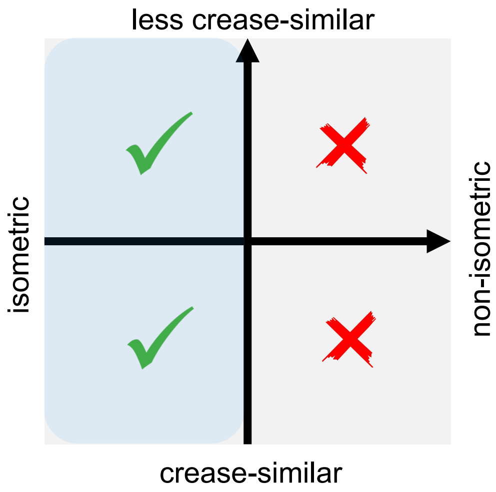
Challenge



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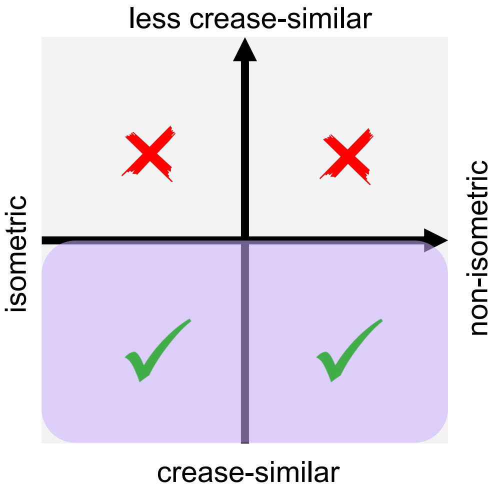

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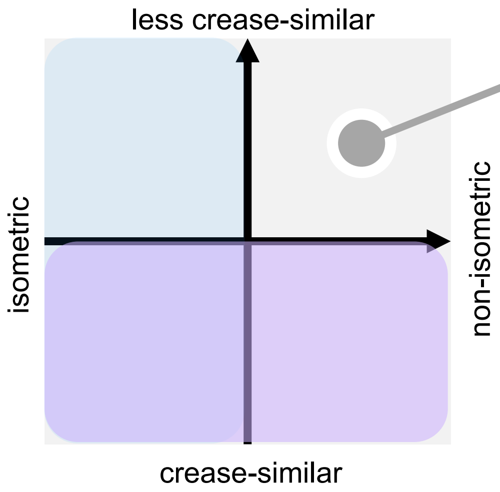
Challenge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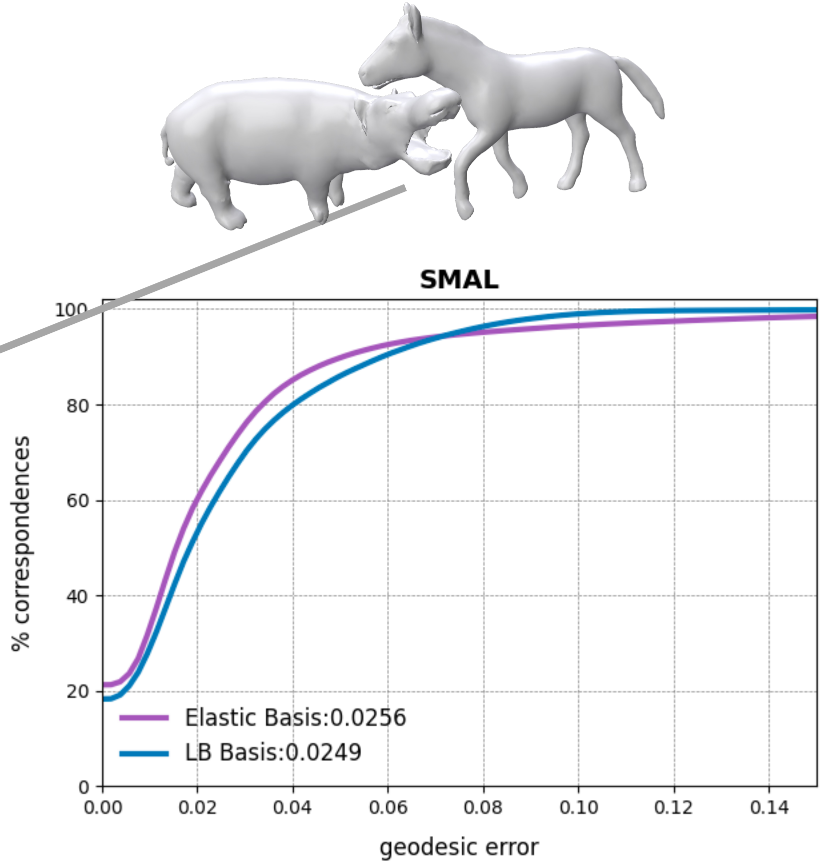
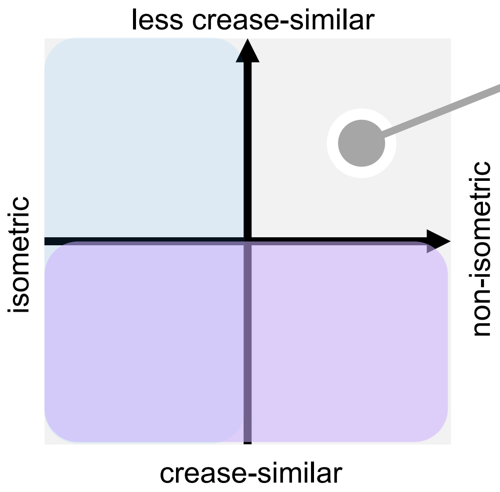
isometric
non-isometric
crease-similar


SMAL
% correspondences
geodesic error
Elastic Basis:0.0256
LB Basis:0.0249
0
20
40
60
80
100
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14

Challenge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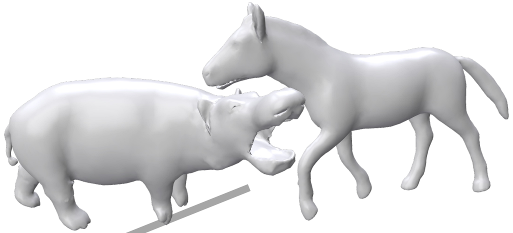

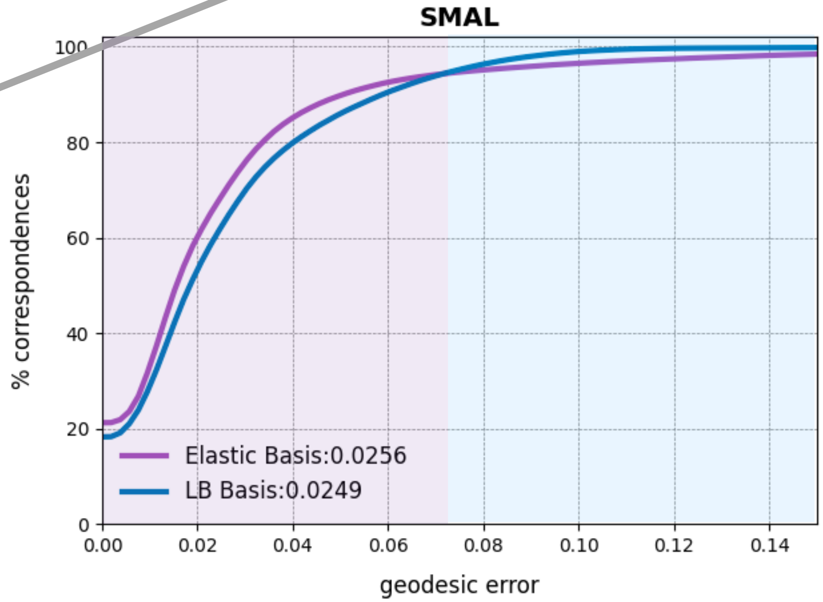
SMAL
% correspondences
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Elastic Basis:0.0256
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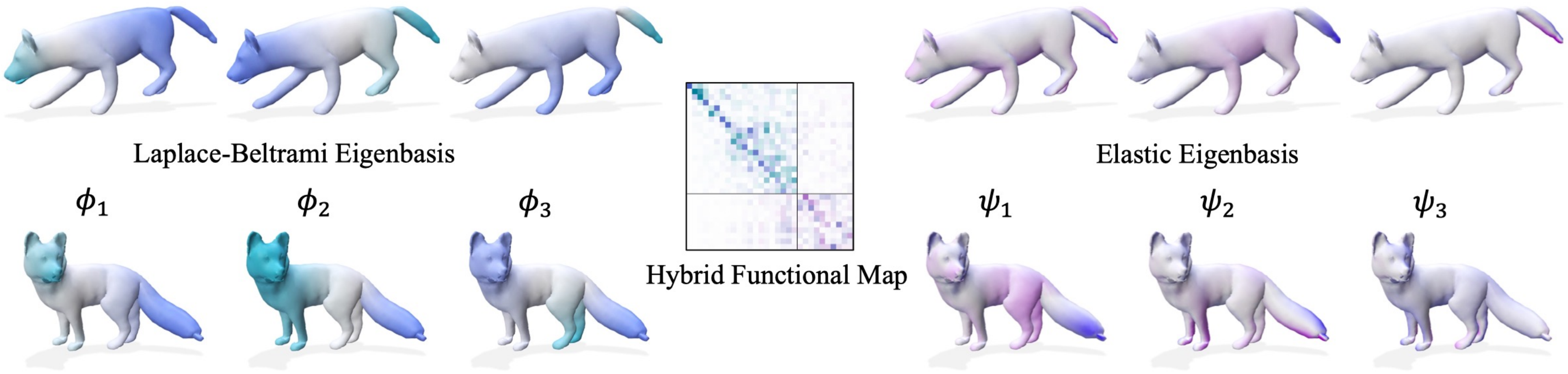
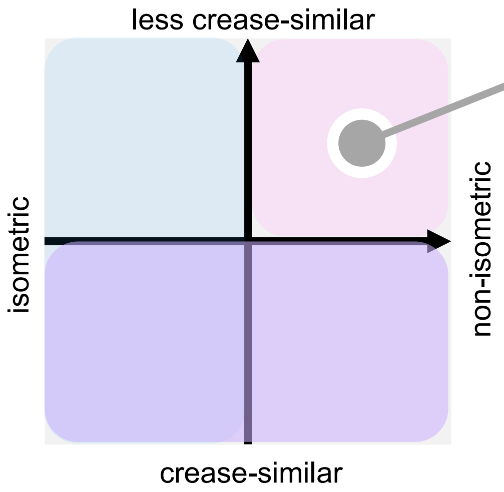
0
20
40
60
80
100
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14

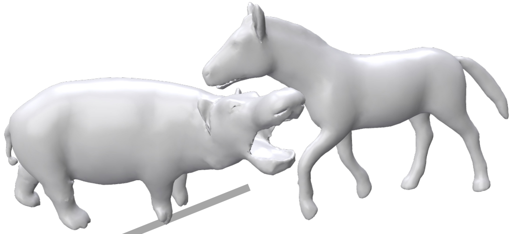
Solution

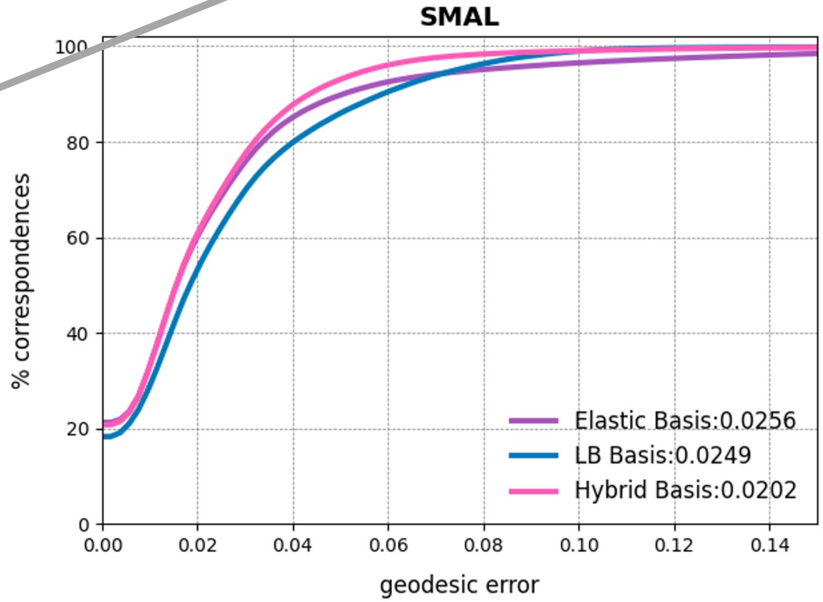


Laplace-Beltrami Eigenbasis
ϕ_1
ϕ_2
ϕ_3
Hybrid Functional Map
Elastic Eigenbasis
ψ_1
ψ_2
ψ_3

Challenge

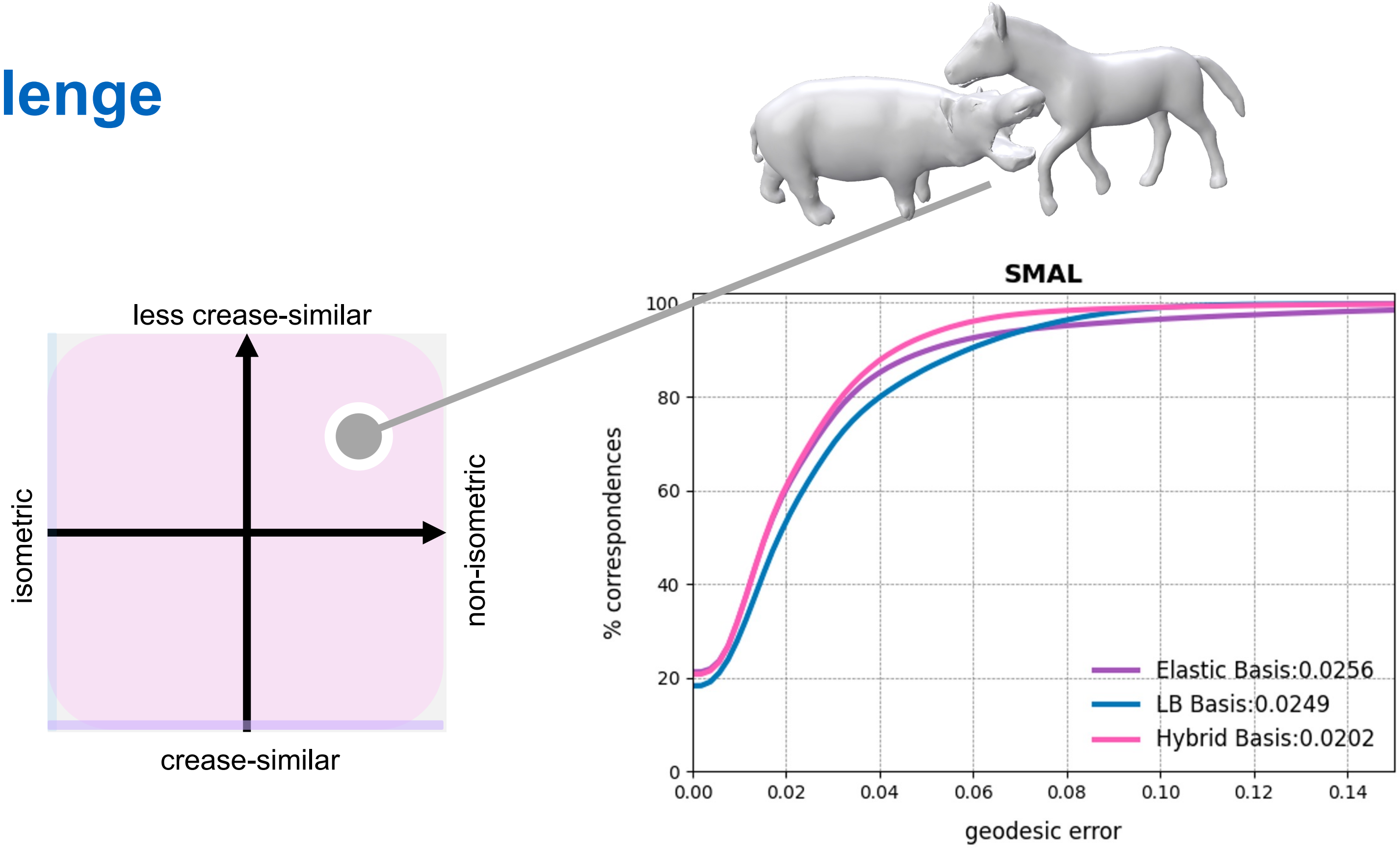






TODO:GT Reconst title:
60 vs 60 vs 40_20basis

Challenge



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SMAL
100
80
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% correspondences
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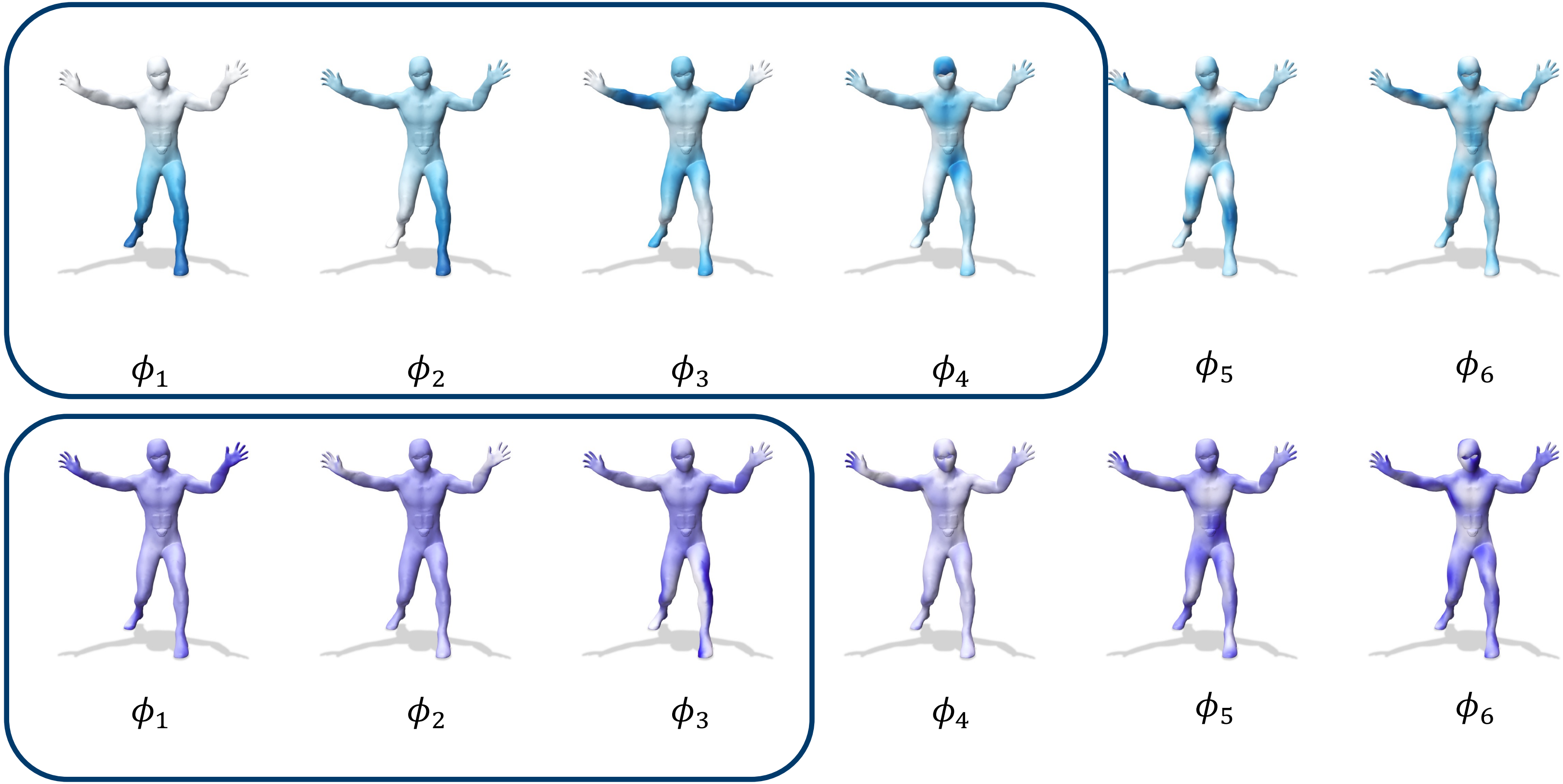
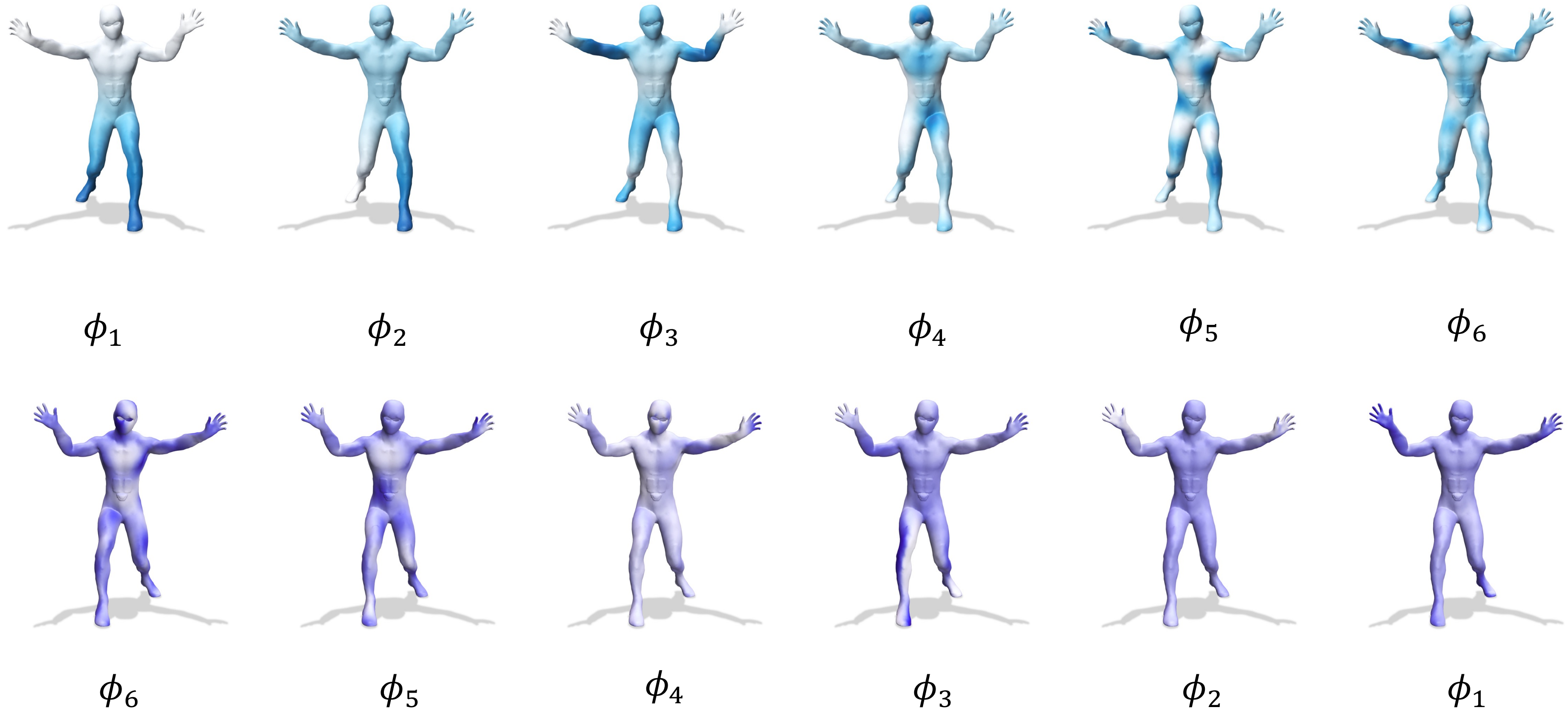
0.02
0.04
0.06
0.08
0.10
0.12
0.14
geodesic error
Elastic Basis:0.0256
LB Basis:0.0249
Hybrid Basis:0.0202

Choice of Hybrid Basis

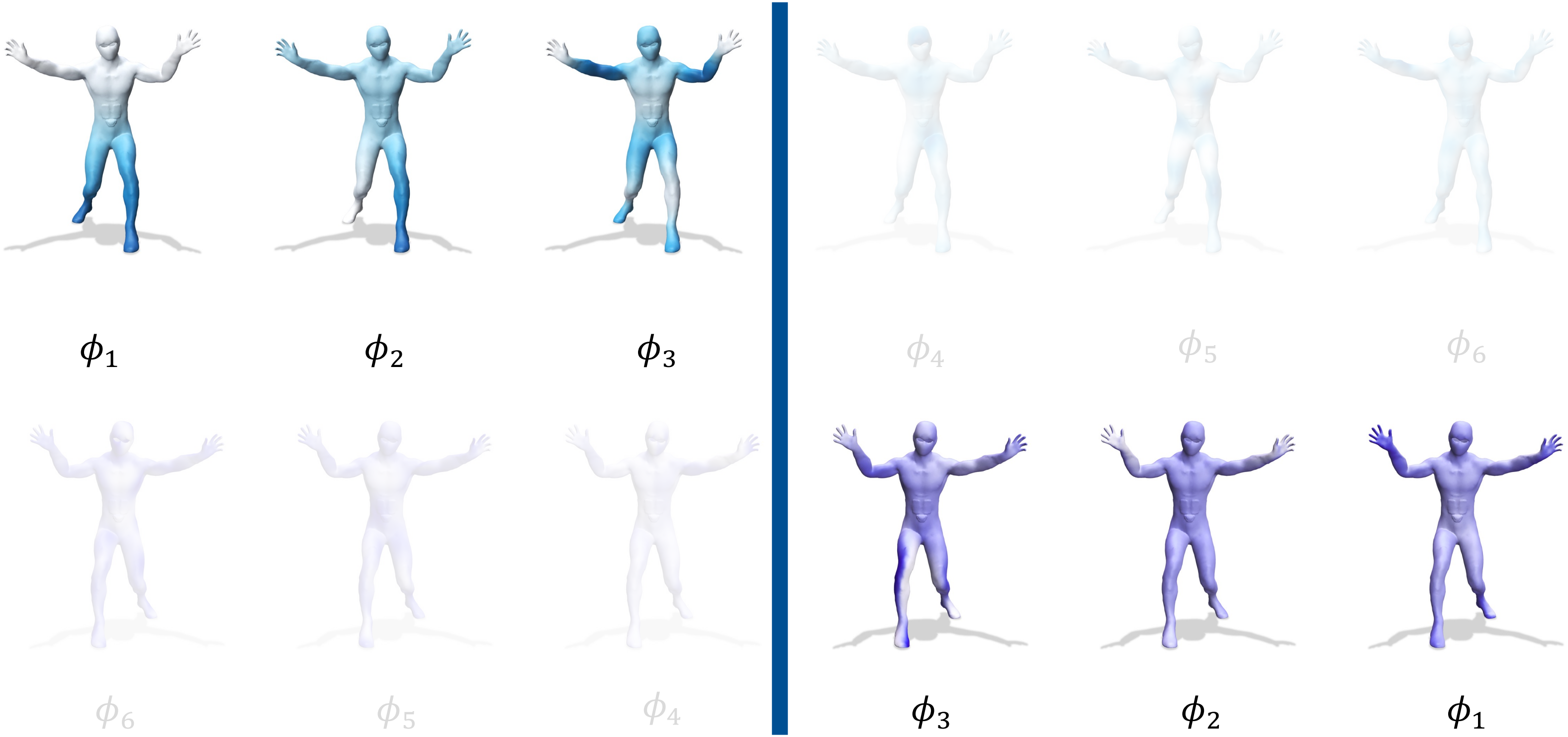
Top **k** Basis



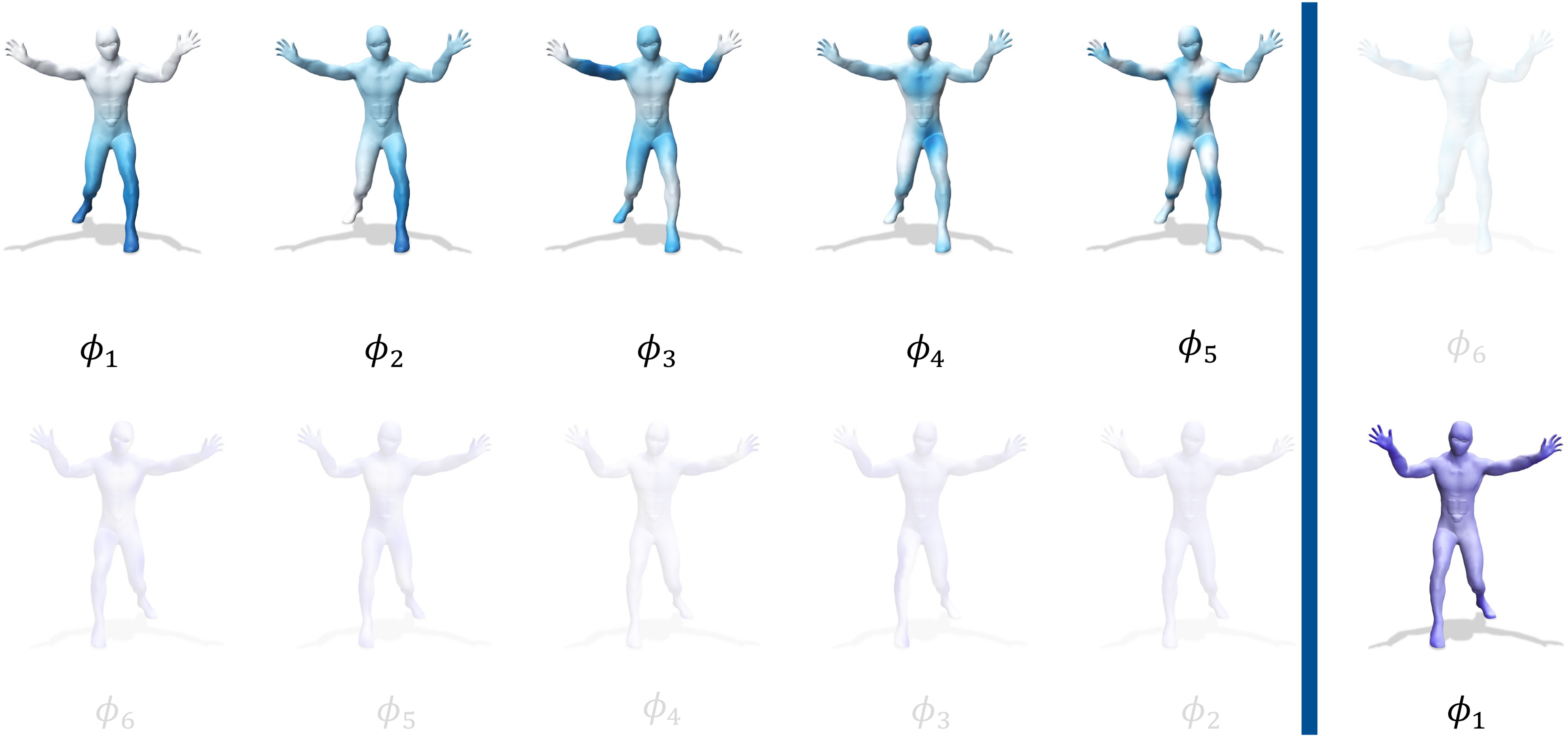
Choice of Hybrid Basis



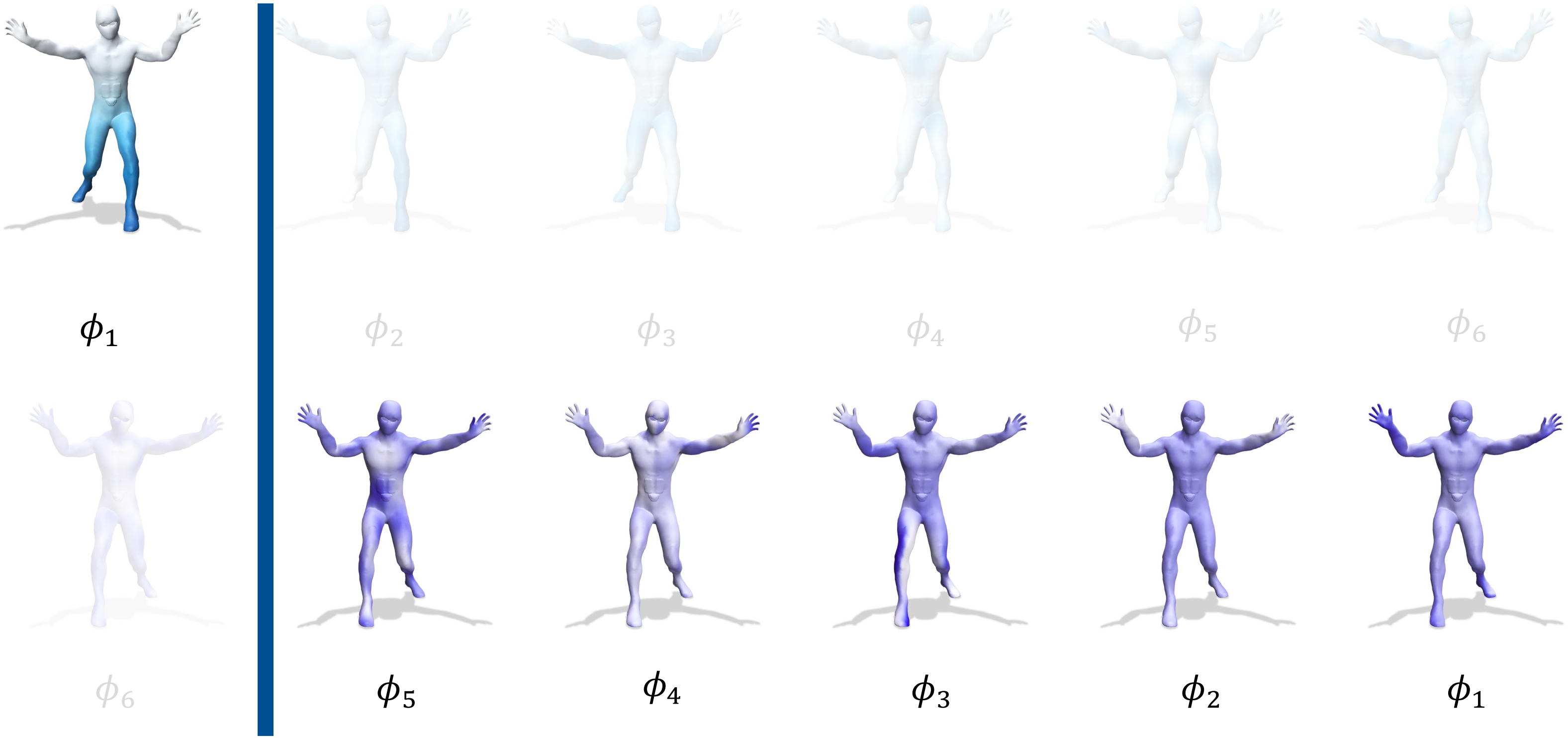
Choice of Hybrid Basis



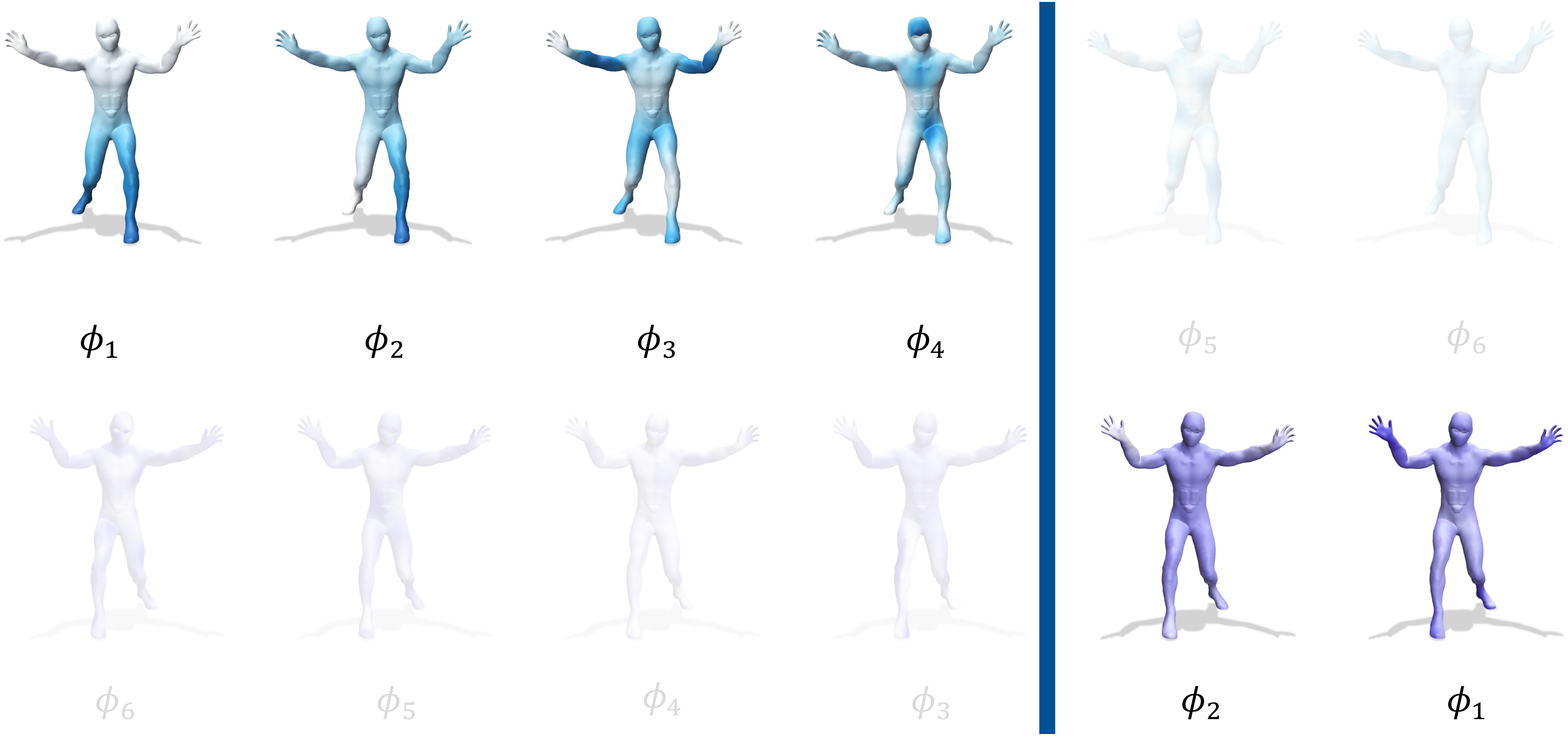
Choice of Hybrid Basis



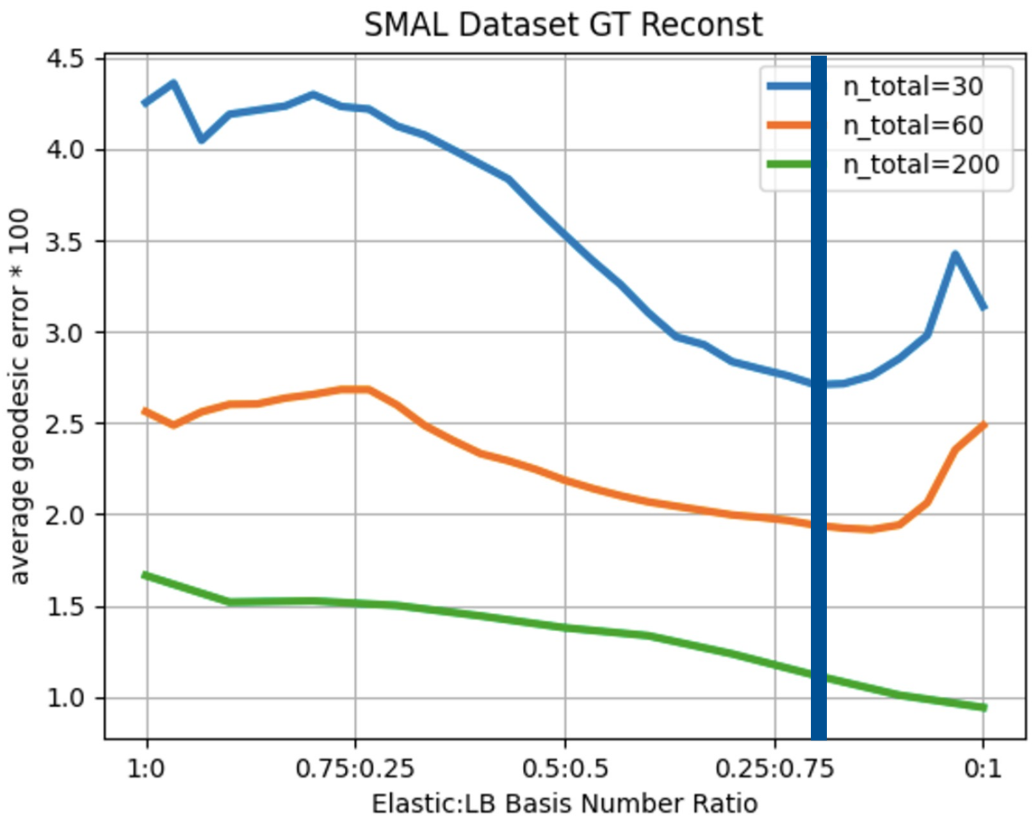
Choice of Hybrid Basis



Choice of Hybrid Basis



Choice of Hybrid Basis

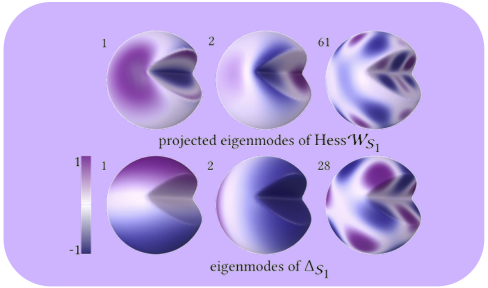


This provides some intuition

But GT Reconst doesn't tell a full story

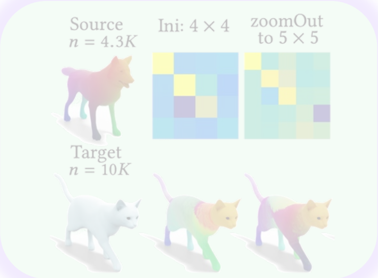
Can it work in the learned setting?

- **Generalized FMap Framework**



An Elastic Basis [Hartwig et al. 2023]

Deep Functional Map Methods



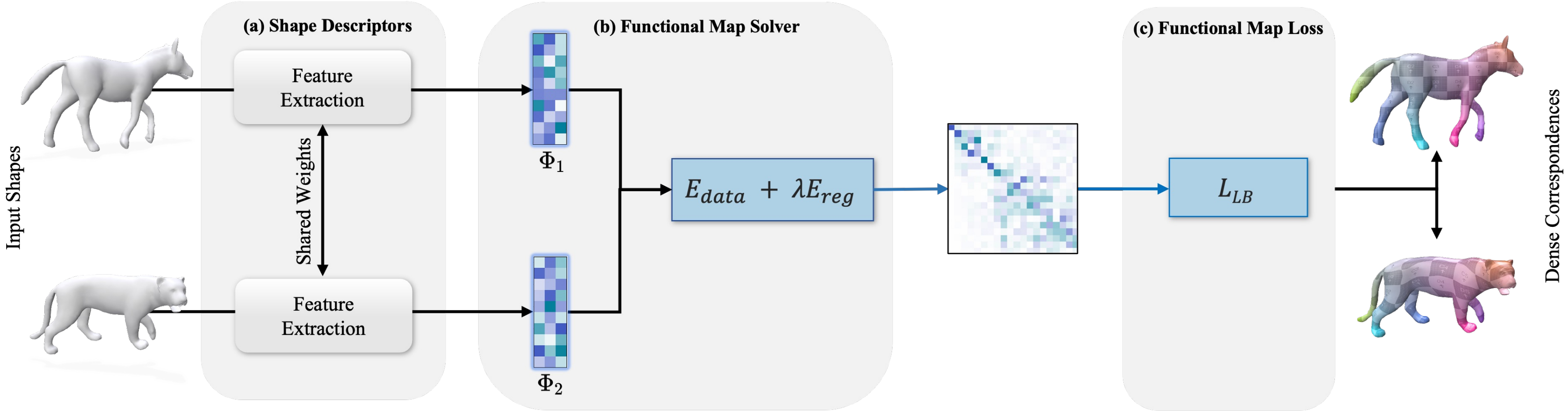
ZoomOut [Melzi et al. 2019]

Smooth Shells [Eisenberger et al. 2020]

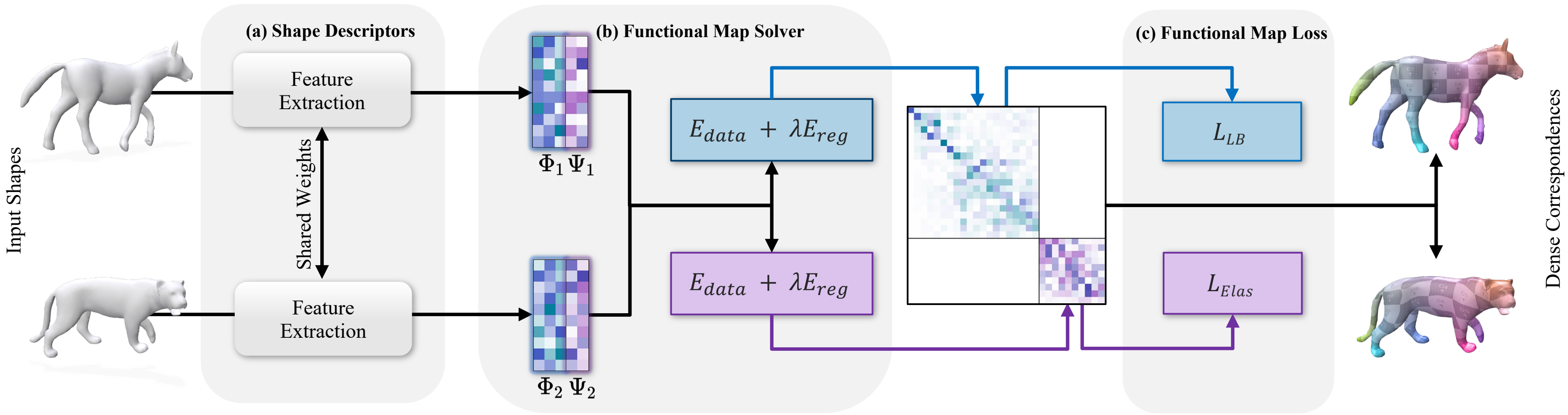
GeomFmaps [Donati et al. 2020]

ULRSSM [Cao et al. 2023]

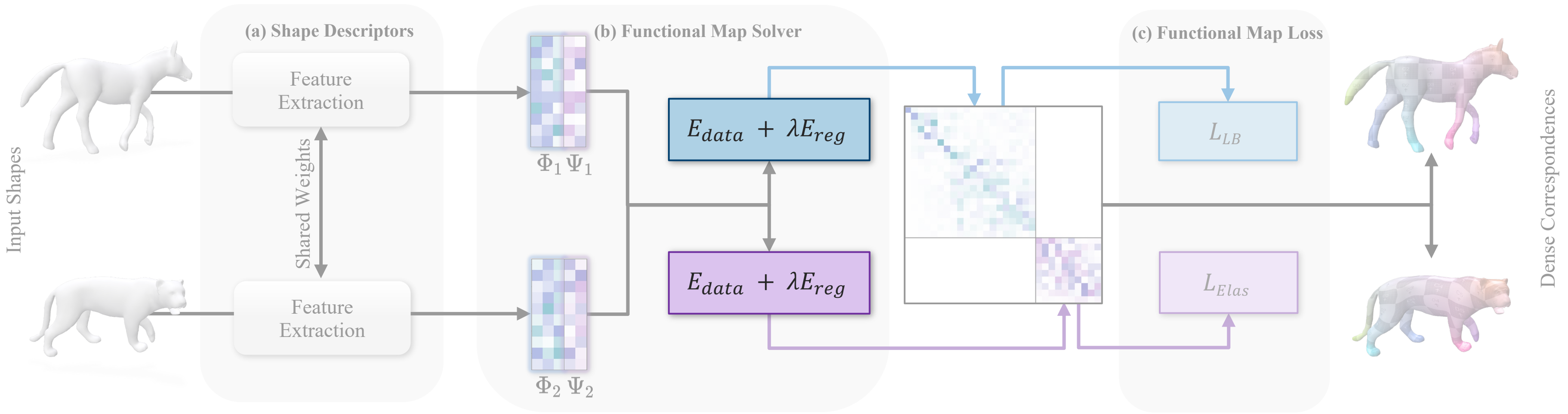
Classical Deep Functional Map Pipeline



Hybrid Deep Functional Map Pipeline



Regularized Functional Map Solver



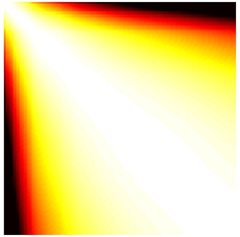
Regularized Functional Map Solver

$$C^* = \arg\min_C E(C) = E_{\text{data}}(C) + \lambda E_{\text{reg}}(C)$$
$$E_{\text{data}}(C) = \|CD_{\Phi_1} - D_{\Phi_2}\|_F^2$$
$$E_{\text{reg}}(C) = \|C\Lambda_1 - \Lambda_2 C\|_F^2$$

$$C^* = \arg\min_C E(C) = E_{\text{data}}(C) + \lambda E_{\text{reg}}(C)$$
$$E_{\text{data}}(C) = \|CD_{\Psi_1} - D_{\Psi_2}\|_{M_{k,2}}^2$$
$$E_{\text{reg}}(C) = \|C\Lambda_1 - \Lambda_2 C\|_{HS}^2$$

Linear Operators Commutativity:
Standard Laplacian or Resolvent

Standard Laplacian.



Resolvent

Regularized Functional Map Solver

$$C^* = \arg\min_C E(C) = E_{\text{data}}(C) + \lambda E_{\text{reg}}(C)$$
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$$E_{\text{reg}}(C) = \|C\Lambda_1 - \Lambda_2 C\|_{HS}^2$$

$$A = D_{\Phi_1}$$

$$B = D_{\Phi_2}$$

$$\Delta_{ij} = (\Lambda_1(j) - \Lambda_2(i))^2$$

$$CAA^T + \lambda\Delta \cdot C = BA^T$$

$$(AA^T + \lambda \text{diag}(\Lambda_1(j) - \Lambda_2(i))^2)c_i = Ab_i$$

Solve for C row wise, result in solving k times $k \times k$ linear system

Regularized Functional Map Solver

$$C^* = \arg\min_C E(C) = E_{\text{data}}(C) + \lambda E_{\text{reg}}(C)$$
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$$E_{\text{reg}}(C) = \|C\Lambda_1 - \Lambda_2 C\|_{HS}^2$$

$$\|CD_{\Psi_1} - D_{\Psi_2}\|_{M_{k,2}} = \|\sqrt{M_{k,2}}(CD_{\Psi_1} - D_{\Psi_2})\|_F$$

$$\|C\Lambda_1 - \Lambda_2 C\|_{HS} = \|\sqrt{M_{k,2}}(C\Lambda_1 - \Lambda_2 C)\sqrt{M_{k,1}^{-1}}\|_F$$

$$\text{vec}(ABC) = (C^\top \otimes A)\,\text{vec}(B)$$

$$\begin{aligned}
&\|\sqrt{M_{k,2}}(CD_{\Psi_1} - D_{\Psi_2})\|_F \\
&= \|vec(\sqrt{M_{k,2}}CD_{\Psi_1}) - vec(\sqrt{M_{k,2}}D_{\Psi_2}))\|_2 \\
&= \|((\sqrt{M_{k,2}}D_{\Psi_1})^\top \otimes I)vec(C) - vec(\sqrt{M_{k,2}}D_{\Psi_2})\|_2
\end{aligned}$$

Regularized Functional Map Solver

$$C^* = \arg\min_C E(C) = E_{\text{data}}(C) + \lambda E_{\text{reg}}(C)$$
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$$\|CD_{\Psi_1} - D_{\Psi_2}\|_{M_{k,2}} = \|\sqrt{M_{k,2}}(CD_{\Psi_1} - D_{\Psi_2})\|_F$$

$$\|C\Lambda_1 - \Lambda_2 C\|_{HS} = \|\sqrt{M_{k,2}}(C\Lambda_1 - \Lambda_2 C)\sqrt{M_{k,1}^{-1}}\|_F$$

$$\text{vec}(ABC) = (C^\top \otimes A)\,\text{vec}(B)$$

$$\|\sqrt{M_{k,2}}\,(C\Lambda_1 - \Lambda_2 C)\,\sqrt{M_{k,1}}\|_{\text{F}}^2$$
$$= \|((\Lambda_1\sqrt{M_{k,1}^{-1}}) \otimes \sqrt{M_{k,2}} -$$
$$\sqrt{M_{k,1}^{-1}} \otimes (\sqrt{M_{k,2}}\Lambda_2))\text{vec}(C)\|_F^2$$

Regularized Functional Map Solver

$$C^* = \arg\min_C E(C) = E_{\text{data}}(C) + \lambda E_{\text{reg}}(C)$$
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$$(AA^T + \lambda \text{diag}(\Lambda_1(j) - \Lambda_2(i))^2)c_i = Ab_i$$

$$A = (\sqrt{M_{k,2}}D_{\Psi_1})^\top \otimes I$$
$$B = \sqrt{M_{k,2}}D_{\Psi_2}$$
$$\zeta = (\Lambda_1\sqrt{M_{k,1}^{-1}}) \otimes \sqrt{M_{k,2}} - \sqrt{M_{k,1}^{-1}} \otimes (\sqrt{M_{k,2}}\Lambda_2)$$

$$(A^\top A + \lambda\zeta^\top\zeta)vec(C) = A^\top vec(B)$$

Regularized Functional Map Solver

$$C^* = \arg\min_C E(C) = E_{\text{data}}(C) + \lambda E_{\text{reg}}(C)$$
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$$C^* = \arg\min_C E(C) = E_{\text{data}}(C) + \lambda E_{\text{reg}}(C)$$
$$E_{\text{data}}(C) = \|CD_{\Psi_1} - D_{\Psi_2}\|_{M_{k,2}}^2$$
$$E_{\text{reg}}(C) = \|C\Lambda_1 - \Lambda_2 C\|_{HS}^2$$

$$A = D_{\Phi_1}$$
$$B = D_{\Phi_2}$$
$$\Delta_{ij} = (\Lambda_1(j) - \Lambda_2(i))^2$$
$$CAA^T + \lambda\Delta \cdot C = BA^T$$
$$(AA^T + \lambda \text{diag}(\Lambda_1(j) - \Lambda_2(i))^2)c_i = Ab_i$$

$$A = (\sqrt{M_{k,2}}D_{\Psi_1})^\top \otimes I$$
$$B = \sqrt{M_{k,2}}D_{\Psi_2}$$
$$\zeta = (\Lambda_1\sqrt{M_{k,1}^{-1}}) \otimes \sqrt{M_{k,2}} - \sqrt{M_{k,1}^{-1}} \otimes (\sqrt{M_{k,2}}\Lambda_2)$$
$$(A^\top A + \lambda\zeta^\top\zeta)vec(C) = A^\top vec(B)$$

Solve for C using Kronecker product and vectorizations, results in a single $k^2 \times k^2$ linear system

Regularized Functional Map Solver

$$C^* = \arg\min_C E(C) = E_{\text{data}}(C) + \lambda E_{\text{reg}}(C)$$
$$E_{\text{data}}(C) = \|CD_{\Phi_1} - D_{\Phi_2}\|_F^2$$
$$E_{\text{reg}}(C) = \|C\Lambda_1 - \Lambda_2 C\|_F^2$$

$$C^* = \arg\min_C E(C) = E_{\text{data}}(C) + \lambda E_{\text{reg}}(C)$$
$$E_{\text{data}}(C) = \|CD_{\Psi_1} - D_{\Psi_2}\|_{M_{k,2}}^2$$
$$E_{\text{reg}}(C) = \|C\Lambda_1 - \Lambda_2 C\|_{HS}^2$$

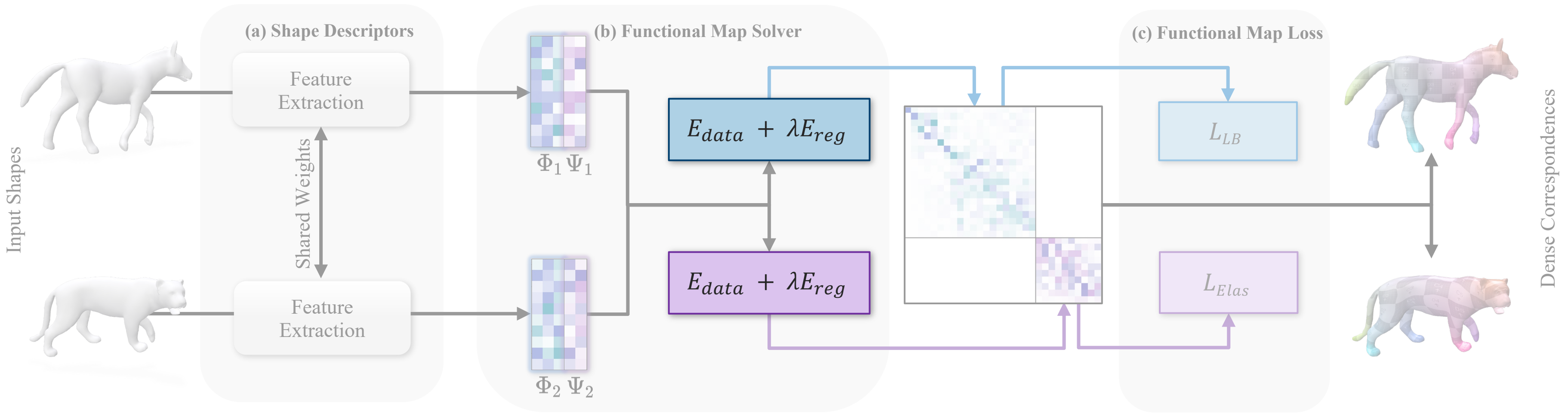
Solve for C row wise, result in solving k times $k \times k$ linear system

Solve for C using Kronecker product and vectorizations, results in a single $k^2 \times k^2$ linear system

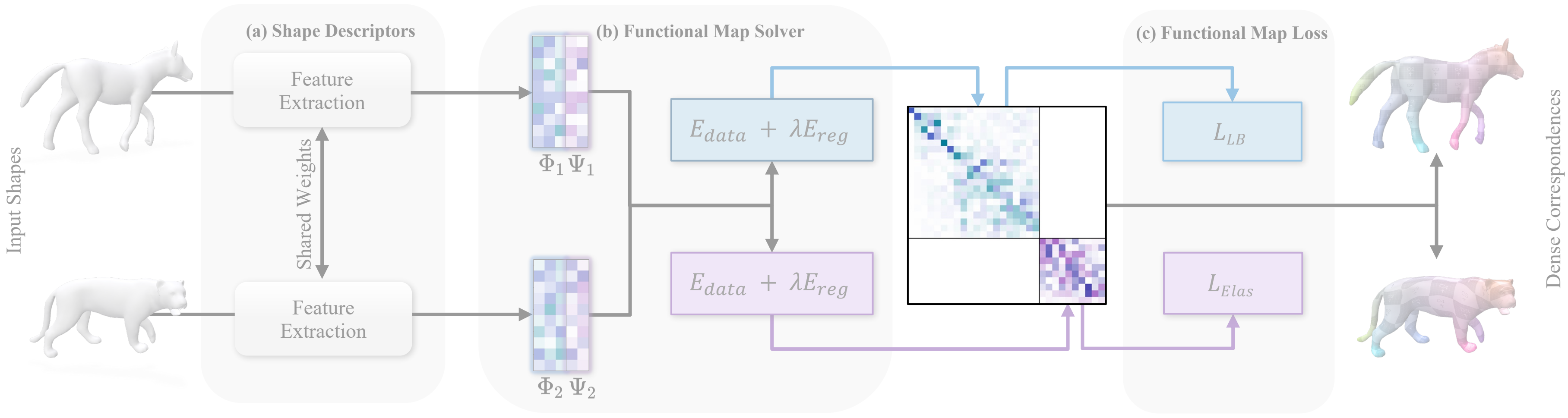
For $k < 100$, this is still feasible

For $k > 100$, prohibitively expensive

Regularized Functional Map Solver



Map Block Separation

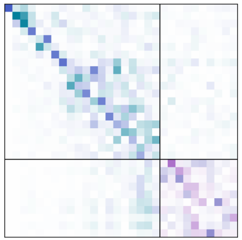


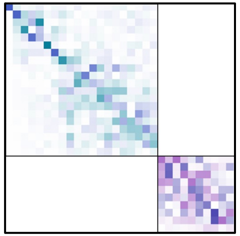
Motivations:

Map Block Separation

Motivations:

1. Regularization

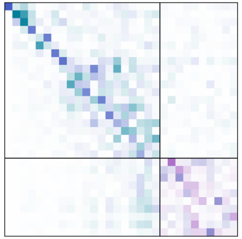


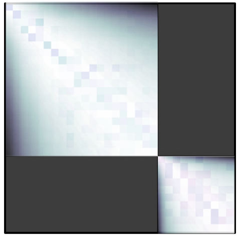


Map Block Separation

Motivations:

1. Regularization

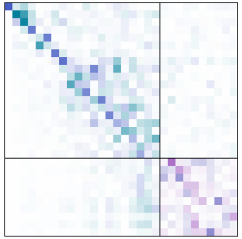


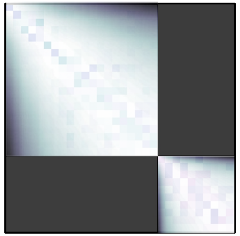


Map Block Separation

Motivations:

1. Regularization





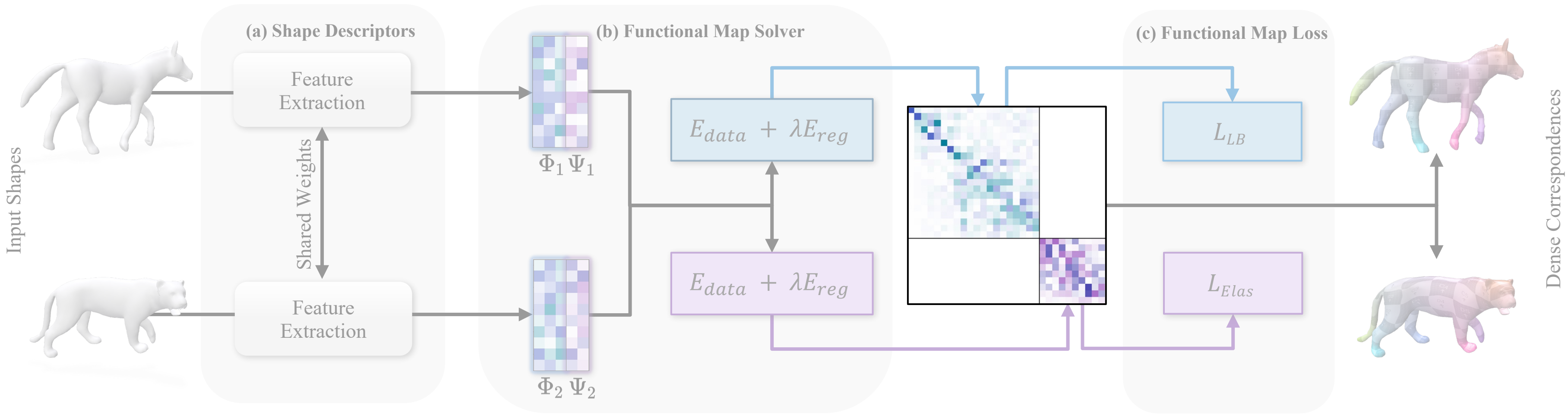
2. Computation

Solve for C using Kronecker product and vectorizations, results in a single $k^2 \times k^2$ linear system

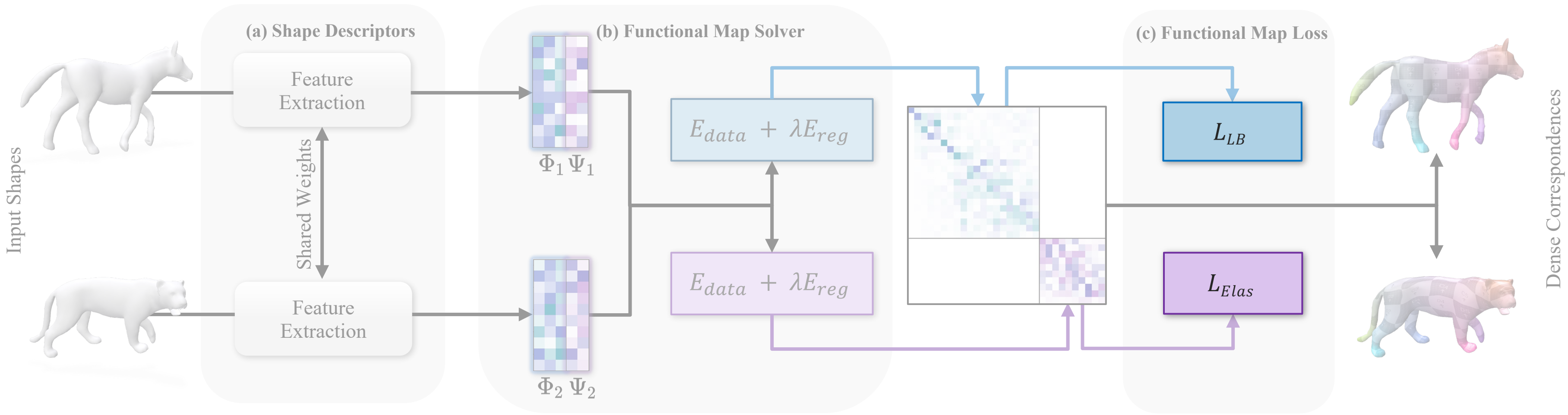
For $k < 100$, this is feasible

For $k > 100$, prohibitively expensive

Map Block Separation



Functional Map Loss



Functional Map Loss

$$\mathcal{L}_{\text{LB}} = \|C - C_{\text{gt}}\|_{\text{F}}^2$$

$$\begin{aligned}\mathcal{L}_{\text{Elas}} &= \|C - C_{\text{gt}}\|_{\text{HS}}^2 \\ &= \|\sqrt{M_{k,2}}(C - C_{\text{gt}})\sqrt{M_{k,1}^{-1}}\|_F^2\end{aligned}$$

GeomFmaps [Donati et al. 2020]

Donati, Nicolas, Abhishek Sharma, and Maks Ovsjanikov. "Deep geometric functional maps: Robust feature learning for shape correspondence." *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*. 2020.

Functional Map Loss

$$\mathcal{L}_{\text{orth}} = \|C_{12}^{*}C_{12} - I\|_F^2 + \|C_{21}^{*}C_{21} - I\|_F^2$$
$$L_{\text{bij}} = \|C_{12}C_{21} - I\|_F^2 + \|C_{21}C_{12} - I\|_F^2$$
$$\mathcal{L}_{\text{couple}} = \left\|C_{12} - \Phi_2^{\dagger}\Pi_{21}\Phi_1\right\|_{\text{F}}^2 + \left\|C_{21} - \Phi_1^{\dagger}\Pi_{12}\Phi_2\right\|_{\text{F}}^2$$

$$\mathcal{L}_{\text{orth}} = \|C_{12}'^{*}C_{12} - I\|_{HS}^2 + \|C_{21}'^{*}C_{21} - I\|_{HS}^2$$
$$L_{\text{bij}} = \|C_{12}C_{21} - I\|_{HS}^2 + \|C_{21}C_{12} - I\|_{HS}^2$$
$$\mathcal{L}_{\text{couple}} = \left\|C_{12} - \Psi_2^{\dagger}\Pi_{21}\Psi_1\right\|_{\text{HS}}^2 + \left\|C_{21} - \Psi_1^{\dagger}\Pi_{12}\Psi_2\right\|_{\text{HS}}^2$$

ULRSSM [Cao et al. 2023]

Cao, D., Roetzer, P., & Bernard, F. (2023). Unsupervised learning of robust spectral shape matching. ACM Transactions on Graphics (TOG). https://doi.org/10.1145/3592107

Functional Map Loss

$$\mathcal{L}_{\text{orth}} = \|C_{12}^{*}C_{12} - I\|_F^2 + \|C_{21}^{*}C_{21} - I\|_F^2$$
$$L_{\text{bij}} = \|C_{12}C_{21} - I\|_F^2 + \|C_{21}C_{12} - I\|_F^2$$
$$\mathcal{L}_{\text{couple}} = \left\|C_{12} - \Phi_2^{\dagger}\Pi_{21}\Phi_1\right\|_{\text{F}}^2 + \left\|C_{21} - \Phi_1^{\dagger}\Pi_{12}\Phi_2\right\|_{\text{F}}^2$$

$$\mathcal{L}_{\text{orth}} = \|C_{21}^{*}C_{21} - I\|_F^2 + \|C_{12}^{*}C_{12} - I\|_F^2$$
$$L_{\text{bij}} = \|C_{12}C_{21} - I\|_F^2 + \|C_{21}C_{12} - I\|_F^2$$
$$\mathcal{L}_{\text{couple}} = \left\|\sqrt{M_{k,2}}(C_{12} - \Psi_2^{\dagger}\Pi_{21}\Psi_1)\sqrt{M_{k,1}^{-1}}\right\|_F^2 + \left\|\sqrt{M_{k,1}}(C_{21} - \Psi_1^{\dagger}\Pi_{12}\Psi_2)\sqrt{M_{k,2}^{-1}}\right\|_F^2$$

ULRSSM [Cao et al. 2023]

Cao, D., Roetzer, P., & Bernard, F. (2023). Unsupervised learning of robust spectral shape matching. ACM Transactions on Graphics (TOG). https://doi.org/10.1145/3592107

Implementation Details

Final Total Loss:

$$\mathcal{L}_{\text{total}} = \alpha\mathcal{L}_{\text{LB}} + \mu\,\beta\,\mathcal{L}_{\text{Elas}}$$

$$\alpha = \frac{1}{2} \cdot \frac{k^2}{(k-l)^2} \qquad \beta = \frac{1}{2} \cdot \frac{k^2}{l^2}$$

- Normalizing Factors

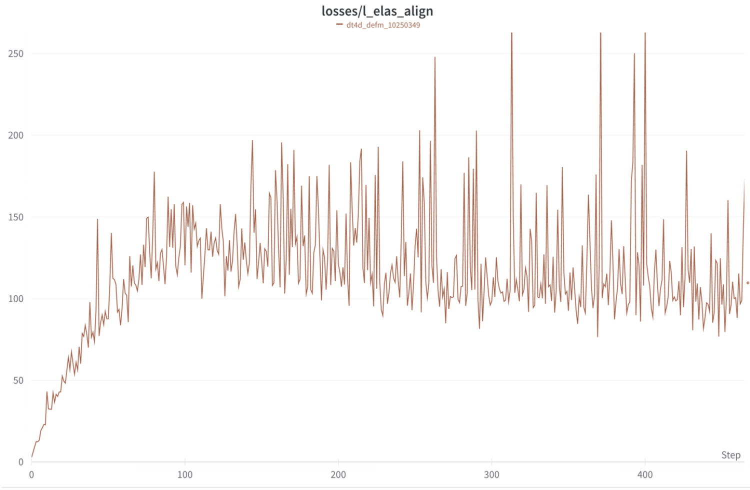
Implementation Details

Final Total Loss:

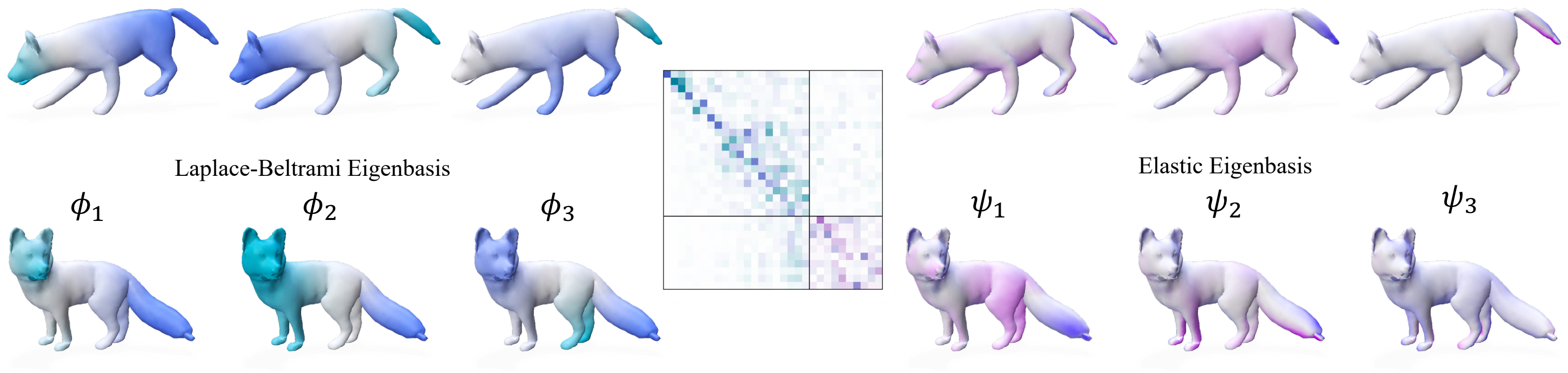
$$\mathcal{L}_{\text{total}} = \alpha \mathcal{L}_{\text{LB}} + \mu\ \beta\ \mathcal{L}_{\text{Elas}}$$

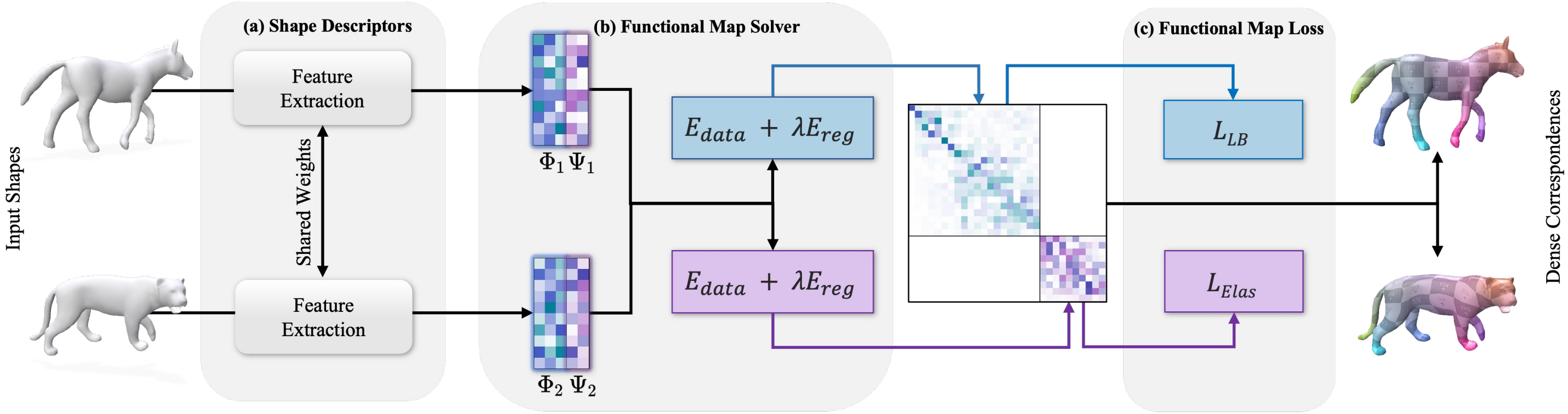
$$\alpha = \frac{1}{2} \cdot \frac{k^2}{(k-l)^2} \qquad \beta = \frac{1}{2} \cdot \frac{k^2}{l^2}$$

- Normalizing Factors
- Additional Linearly increasing Elastic Loss over the first 2000 iters

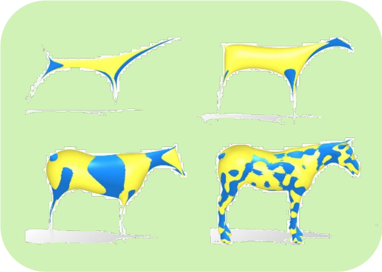


That's everything





Results: it works



Smooth Shells [Eisenberger et al. 2020]

Axiomatic

		FAUST	SCAPE	SHREC'19†	SMAL	DT4D-H intra-class	DT4D-H inter-class	TOPKIDS
Axiomatic	ZoomOut [33]	6.1	7.5	-	38.4	4.0	29.0	33.7
	DiscreteOp [41]	5.6	13.1	-	38.1	3.6	27.6	35.5
	Smooth Shells [16]	**2.5**	4.2	-	30.0	1.1	6.3	10.8
	Hybrid Smooth Shells (ours)	2.6	4.2	-	**28.4**	x	x	**7.5**
Sup.	FMNet [27]	11.0	33.0	-	42.0	9.6	38.0	-
	GeomFMaps [13]	2.6	3.0	7.9	8.4	**2.1**	4.3	-
	Hybrid GeomFMaps (ours)	**2.4**	**2.8**	**5.6**	**7.6**	2.3	**4.2**	-
Unsupervised	Deep Shells [17]	1.7	2.5	21.1	29.3	3.4	31.1	13.7
	DUO-FMNet [15]	2.5	4.2	6.4	6.7	2.6	15.8	-
	AttentiveFMaps-Fast [25]	1.9	2.1	6.3	5.8	1.2	14.6	28.5
	AttentiveFMaps [25]	1.9	2.2	5.8	5.4	1.7	11.6	23.4
	SSCDFM [48]	1.7	2.6	3.8	5.4	1.2	6.1	-
	ULRSSM [10]	1.6	1.9	4.6	3.9	**0.9**	4.1	9.2
	Hybrid ULRSSM (ours)	**1.4**	**1.8**	**4.1**	**3.3**	1.0	**3.5**	**5.1**

Eisenberger, Marvin, Zorah Lahner, and Daniel Cremers. "Smooth shells: Multi-scale shape registration with functional maps." *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*. 2020.

Results: it works

GeomFmaps [Donati et al. 2020]

Supervised Learning

		FAUST	SCAPE	SHREC'19†	SMAL	DT4D-H intra-class	DT4D-H inter-class	TOPKIDS
Axiomatic	ZoomOut [33]	6.1	7.5	-	38.4	4.0	29.0	33.7
	DiscreteOp [41]	5.6	13.1	-	38.1	3.6	27.6	35.5
	Smooth Shells [16]	**2.5**	4.2	-	30.0	1.1	6.3	10.8
	Hybrid Smooth Shells (ours)	2.6	4.2	-	**28.4**	x	x	**7.5**
Sup.	FMNet [27]	11.0	33.0	-	42.0	9.6	38.0	-
	GeomFMaps [13]	2.6	3.0	7.9	8.4	**2.1**	4.3	-
	Hybrid GeomFMaps (ours)	**2.4**	**2.8**	**5.6**	**7.6**	2.3	**4.2**	-
Unsupervised	Deep Shells [17]	1.7	2.5	21.1	29.3	3.4	31.1	13.7
	DUO-FMNet [15]	2.5	4.2	6.4	6.7	2.6	15.8	-
	AttentiveFMaps-Fast [25]	1.9	2.1	6.3	5.8	1.2	14.6	28.5
	AttentiveFMaps [25]	1.9	2.2	5.8	5.4	1.7	11.6	23.4
	SSCDFM [48]	1.7	2.6	3.8	5.4	1.2	6.1	-
	ULRSSM [10]	1.6	1.9	4.6	3.9	**0.9**	4.1	9.2
	Hybrid ULRSSM (ours)	**1.4**	**1.8**	**4.1**	**3.3**	1.0	**3.5**	**5.1**

Donati, Nicolas, Abhishek Sharma, and Maks Ovsjanikov. "Deep geometric functional maps: Robust feature learning for shape correspondence." *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, 2020.

Results: it works

ULRSSM [Cao et al. 2023]

Unsupervised Learning

		FAUST	SCAPE	SHREC'19†	SMAL	DT4D-H intra-class	DT4D-H inter-class	TOPKIDS
Axiomatic	ZoomOut [33]	6.1	7.5	-	38.4	4.0	29.0	33.7
	DiscreteOp [41]	5.6	13.1	-	38.1	3.6	27.6	35.5
	Smooth Shells [16]	**2.5**	4.2	-	30.0	1.1	6.3	10.8
	Hybrid Smooth Shells (ours)	2.6	4.2	-	**28.4**	x	x	**7.5**
Sup.	FMNet [27]	11.0	33.0	-	42.0	9.6	38.0	-
	GeomFMaps [13]	2.6	3.0	7.9	8.4	**2.1**	4.3	-
	Hybrid GeomFMaps (ours)	**2.4**	**2.8**	**5.6**	**7.6**	2.3	**4.2**	-
Unsupervised	Deep Shells [17]	1.7	2.5	21.1	29.3	3.4	31.1	13.7
	DUO-FMNet [15]	2.5	4.2	6.4	6.7	2.6	15.8	-
	AttentiveFMaps-Fast [25]	1.9	2.1	6.3	5.8	1.2	14.6	28.5
	AttentiveFMaps [25]	1.9	2.2	5.8	5.4	1.7	11.6	23.4
	SSCDFM [48]	1.7	2.6	3.8	5.4	1.2	6.1	-
	ULRSSM [10]	1.6	1.9	4.6	3.9	**0.9**	4.1	9.2
	Hybrid ULRSSM (ours)	**1.4**	**1.8**	**4.1**	**3.3**	1.0	**3.5**	**5.1**

Cao, D., Roetzer, P., & Bernard, F. (2023). Unsupervised learning of robust spectral shape matching. ACM Transactions on Graphics (TOG). https://doi.org/10.1145/3592107

Results: it works

Near-Isometric

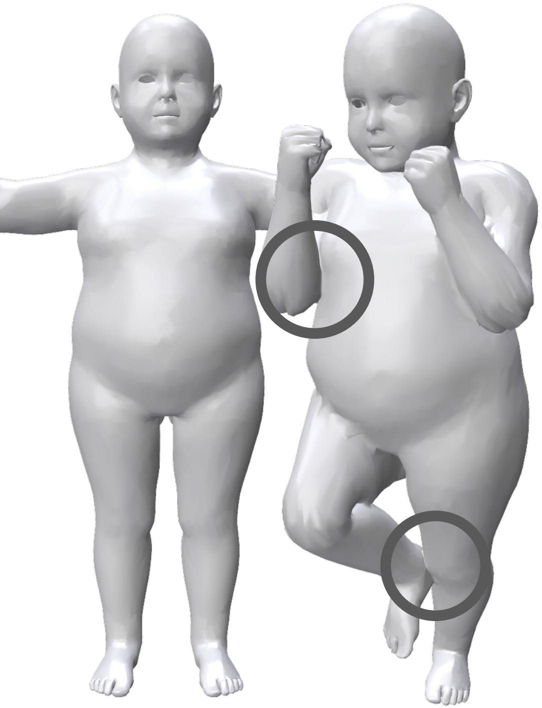
		FAUST	SCAPE	SHREC'19†	SMAL	DT4D-H intra-class	DT4D-H inter-class	TOPKIDS
Axiomatic	ZoomOut [33]	6.1	7.5	-	38.4	4.0	29.0	33.7
	DiscreteOp [41]	5.6	13.1	-	38.1	3.6	27.6	35.5
	Smooth Shells [16]	**2.5**	4.2	-	30.0	1.1	6.3	10.8
	Hybrid Smooth Shells (ours)	2.6	4.2	-	**28.4**	x	x	**7.5**
Sup.	FMNet [27]	11.0	33.0	-	42.0	9.6	38.0	-
	GeomFMaps [13]	2.6	3.0	7.9	8.4	**2.1**	4.3	-
	Hybrid GeomFMaps (ours)	**2.4**	**2.8**	**5.6**	**7.6**	2.3	**4.2**	-
Unsupervised	Deep Shells [17]	1.7	2.5	21.1	29.3	3.4	31.1	13.7
	DUO-FMNet [15]	2.5	4.2	6.4	6.7	2.6	15.8	-
	AttentiveFMaps-Fast [25]	1.9	2.1	6.3	5.8	1.2	14.6	28.5
	AttentiveFMaps [25]	1.9	2.2	5.8	5.4	1.7	11.6	23.4
	SSCDFM [48]	1.7	2.6	3.8	5.4	1.2	6.1	-
	ULRSSM [10]	1.6	1.9	4.6	3.9	**0.9**	4.1	9.2
	Hybrid ULRSSM (ours)	**1.4**	**1.8**	**4.1**	**3.3**	1.0	**3.5**	**5.1**

Results: it works

Non-Isometric

		FAUST	SCAPE	SHREC'19[†]	SMAL	DT4D-H intra-class	DT4D-H inter-class	TOPKIDS
Axiomatic	ZoomOut [33]	6.1	7.5	-	38.4	4.0	29.0	33.7
	DiscreteOp [41]	5.6	13.1	-	38.1	3.6	27.6	35.5
	Smooth Shells [16]	**2.5**	4.2	-	30.0	1.1	6.3	10.8
	Hybrid Smooth Shells (ours)	2.6	4.2	-	**28.4**	x	x	**7.5**
Sup.	FMNet [27]	11.0	33.0	-	42.0	9.6	38.0	-
	GeomFMaps [13]	2.6	3.0	7.9	8.4	**2.1**	4.3	-
	Hybrid GeomFMaps (ours)	**2.4**	**2.8**	**5.6**	**7.6**	2.3	**4.2**	-
Unsupervised	Deep Shells [17]	1.7	2.5	21.1	29.3	3.4	31.1	13.7
	DUO-FMNet [15]	2.5	4.2	6.4	6.7	2.6	15.8	-
	AttentiveFMaps-Fast [25]	1.9	2.1	6.3	5.8	1.2	14.6	28.5
	AttentiveFMaps [25]	1.9	2.2	5.8	5.4	1.7	11.6	23.4
	SSCDFM [48]	1.7	2.6	3.8	5.4	1.2	6.1	-
	ULRSSM [10]	1.6	1.9	4.6	3.9	**0.9**	4.1	9.2
	Hybrid ULRSSM (ours)	**1.4**	**1.8**	**4.1**	**3.3**	1.0	**3.5**	**5.1**

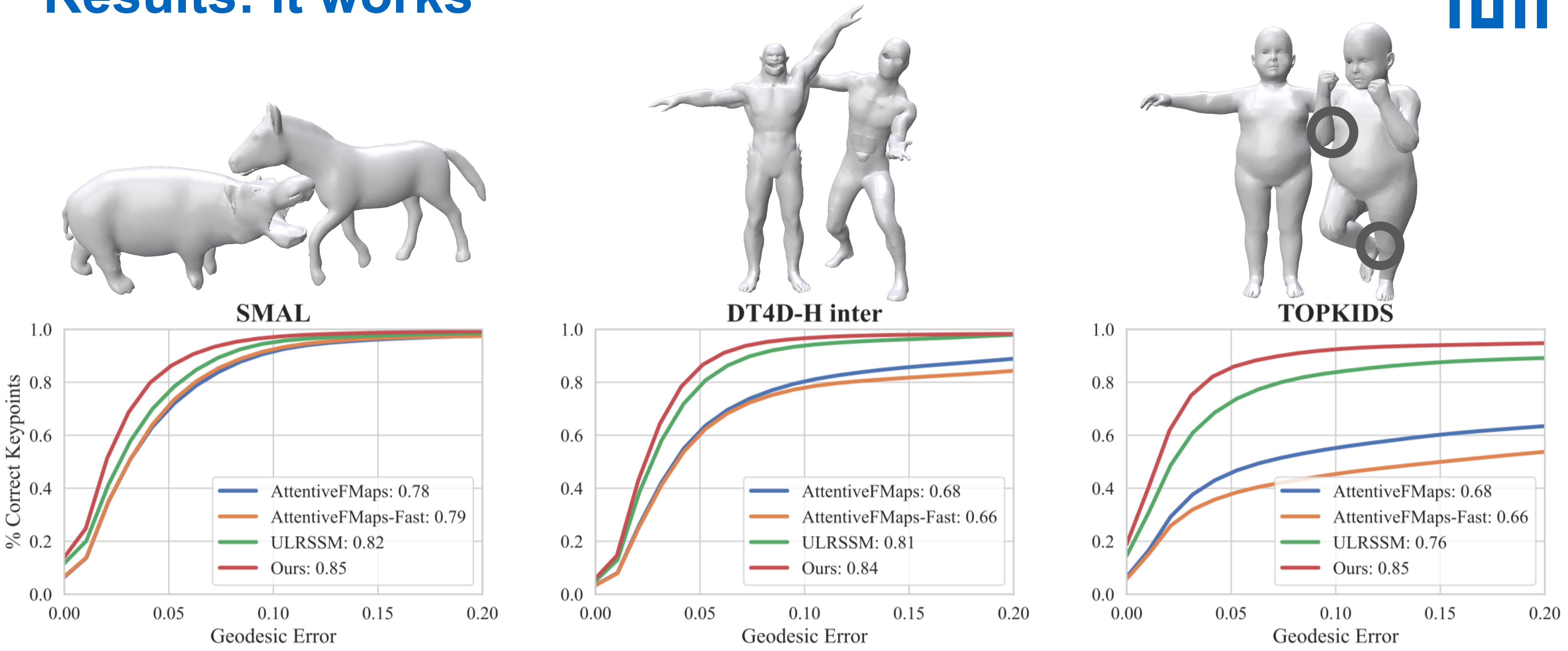
Results: it works



Topologically Noisy

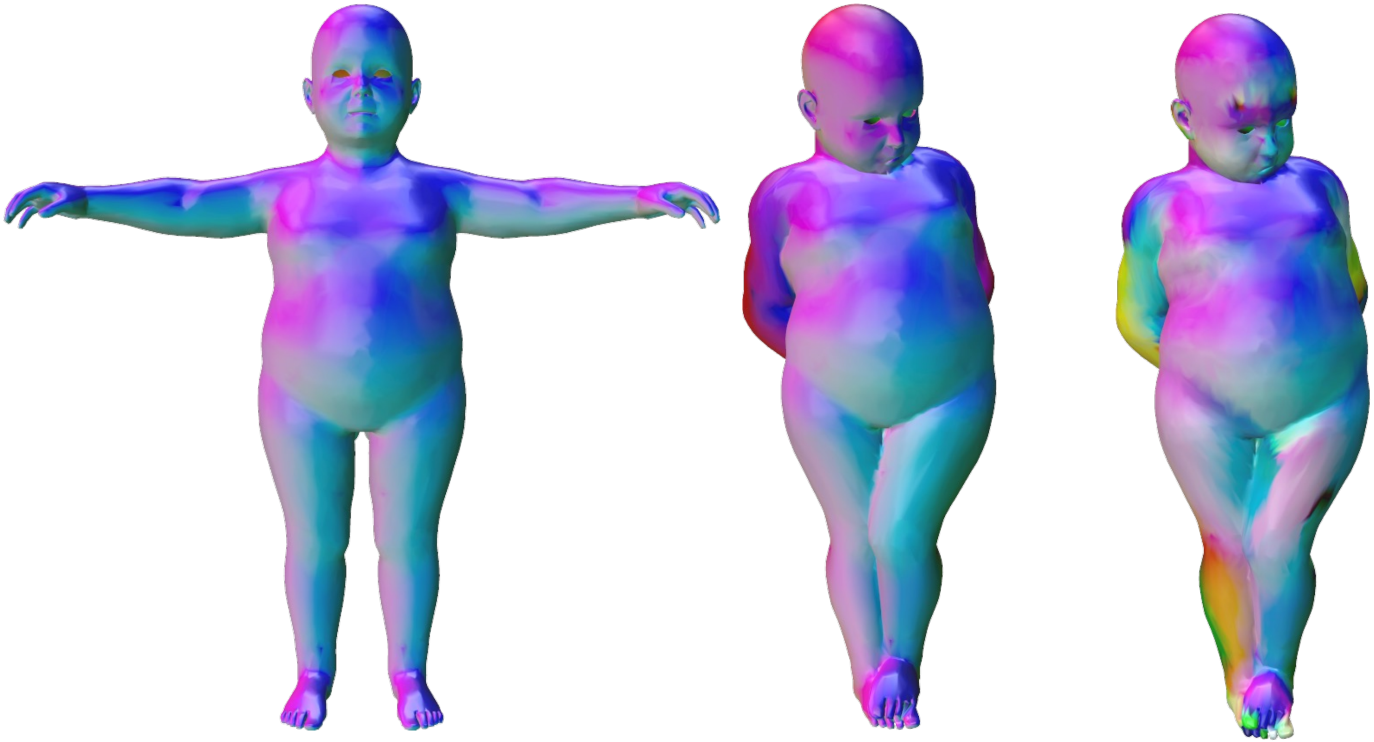
		FAUST	SCAPE	SHREC'19†	SMAL	DT4D-H intra-class	DT4D-H inter-class	TOPKIDS
Axiomatic	ZoomOut [33]	6.1	7.5	-	38.4	4.0	29.0	33.7
	DiscreteOp [41]	5.6	13.1	-	38.1	3.6	27.6	35.5
	Smooth Shells [16]	**2.5**	4.2	-	30.0	1.1	6.3	10.8
	Hybrid Smooth Shells (ours)	2.6	4.2	-	**28.4**	x	x	**7.5**
Sup.	FMNet [27]	11.0	33.0	-	42.0	9.6	38.0	-
	GeomFMaps [13]	2.6	3.0	7.9	8.4	**2.1**	4.3	-
	Hybrid GeomFMaps (ours)	**2.4**	**2.8**	**5.6**	**7.6**	2.3	**4.2**	-
Unsupervised	Deep Shells [17]	1.7	2.5	21.1	29.3	3.4	31.1	13.7
	DUO-FMNet [15]	2.5	4.2	6.4	6.7	2.6	15.8	-
	AttentiveFMaps-Fast [25]	1.9	2.1	6.3	5.8	1.2	14.6	28.5
	AttentiveFMaps [25]	1.9	2.2	5.8	5.4	1.7	11.6	23.4
	SSCDFM [48]	1.7	2.6	3.8	5.4	1.2	6.1	-
	ULRSSM [10]	1.6	1.9	4.6	3.9	**0.9**	4.1	9.2
	Hybrid ULRSSM (ours)	**1.4**	**1.8**	**4.1**	**3.3**	1.0	**3.5**	**5.1**

Results: it works



Results: it works

More Accurate Crease lines alignments



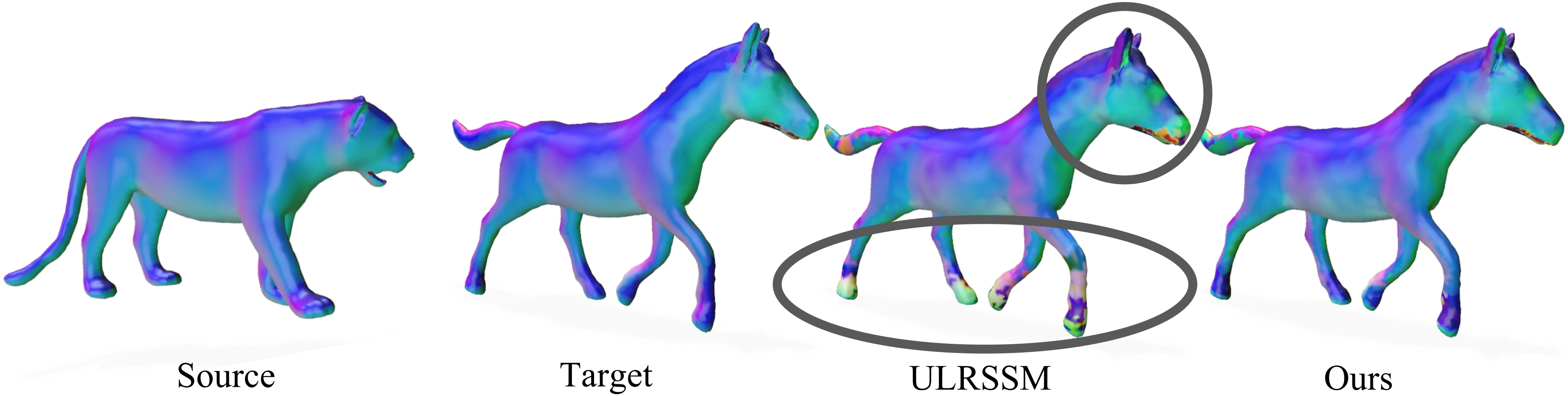
Source Target ULRSSM Ours

Results: it works

Reliable under topological noise

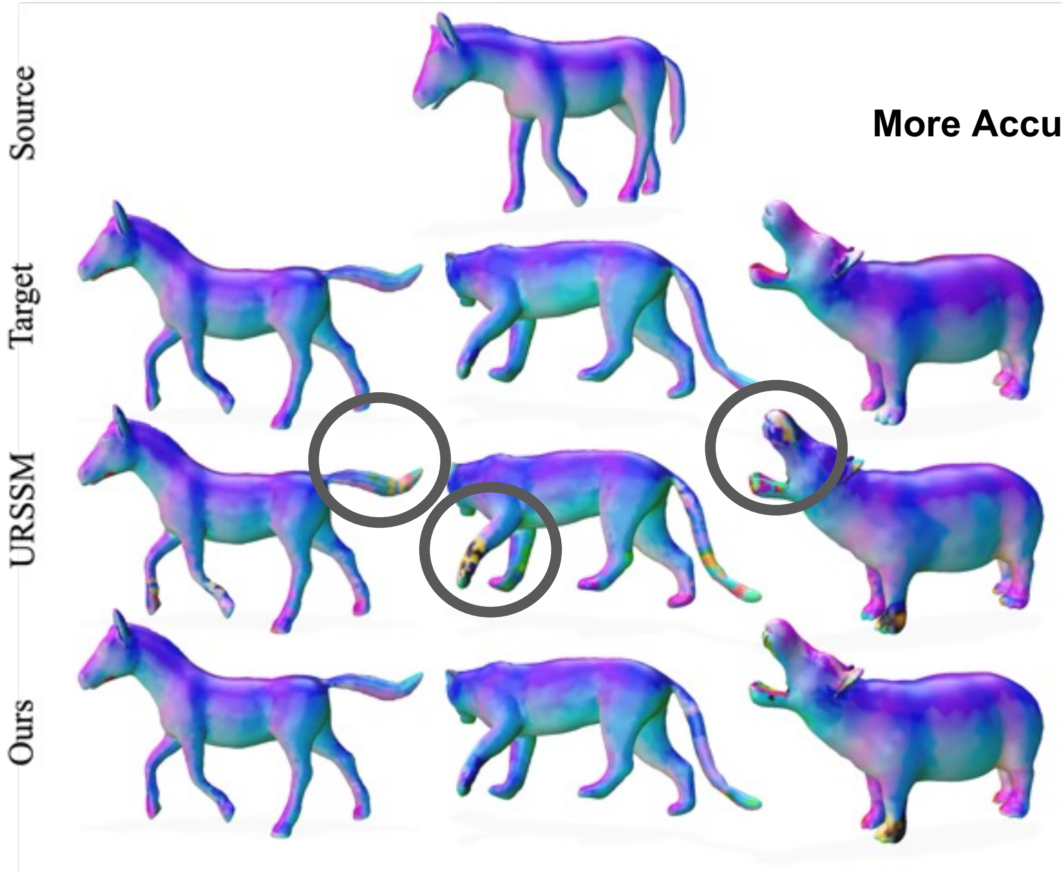
Source Target ULRSSM Ours

Results: it works



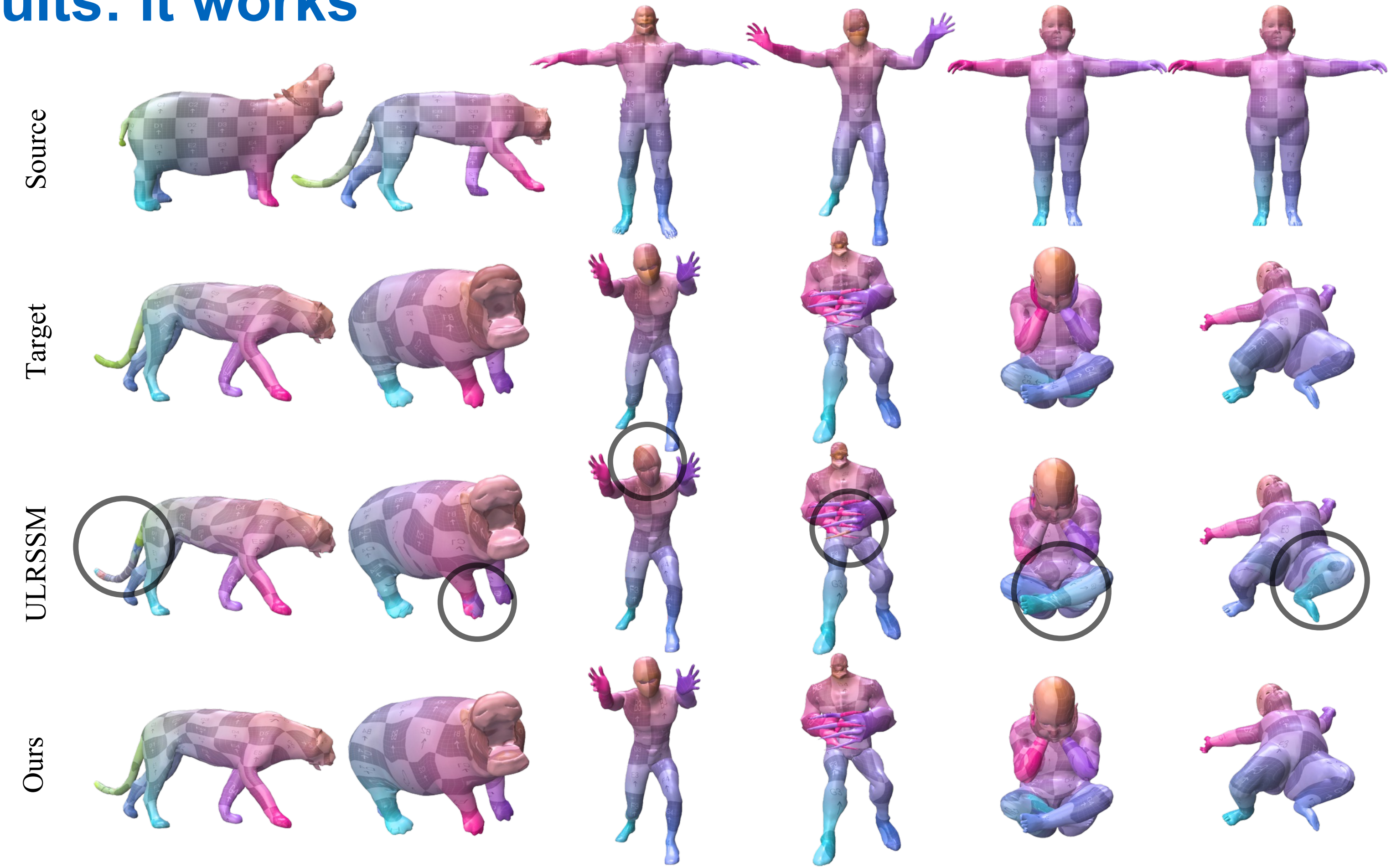
Acurate thin structure of the legs and detail alignments on the face

Results: it works

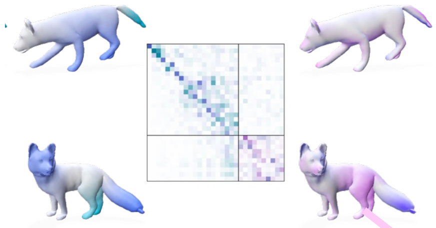


More Accurate Detail Alignments

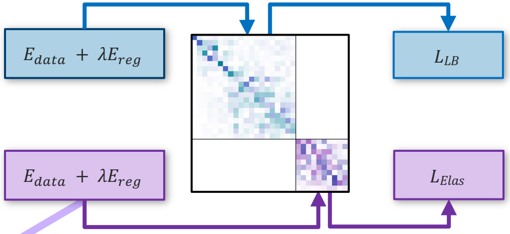
Results: it works



Ablation Studies



Our Hybrid Functional Map

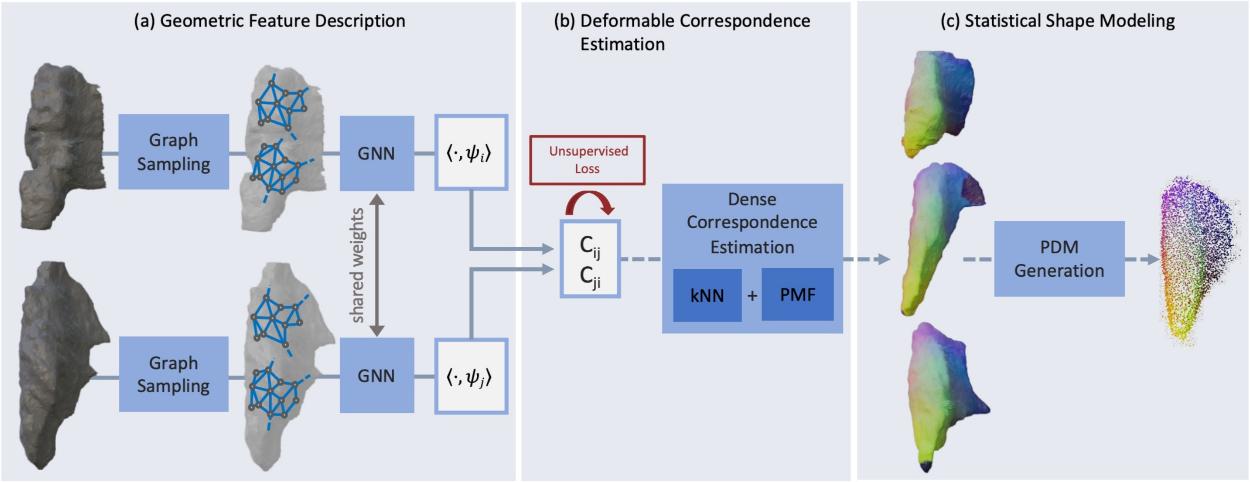


Our Generalized Framework Adaptation

LBO	Elastic	Elastic Stabil.	Geo. error (×100)
✗	✓	✗	40.2 ± 0.80
✗	✓	Orthog.	5.75 ± 1.20
✓	✗	✗	5.15 ± 0.99
✓	✓	✗	4.37 ± 1.57
✓	✓	Orthog.	4.33 ± 0.56
✓	✓	Weight. Norm	$\mathbf{3.83} \pm \mathbf{0.74}$

Ablation table: ULRSSM on SMAL Dataset

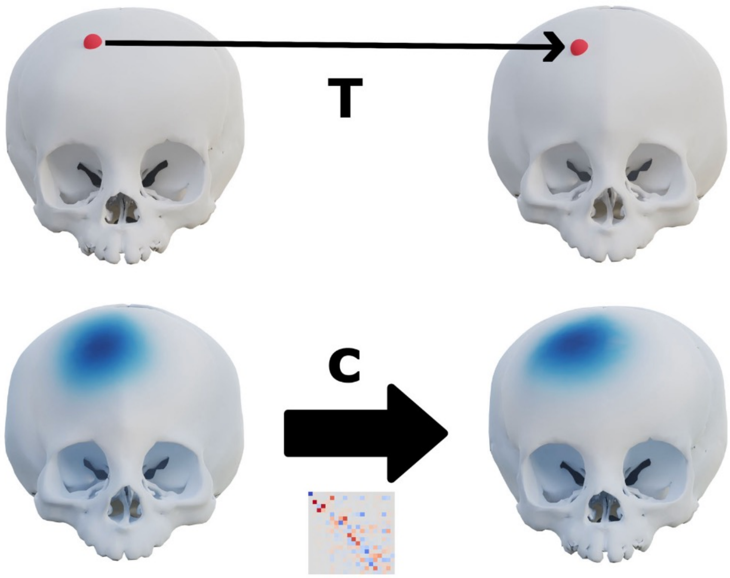
Applications to Medical Domain

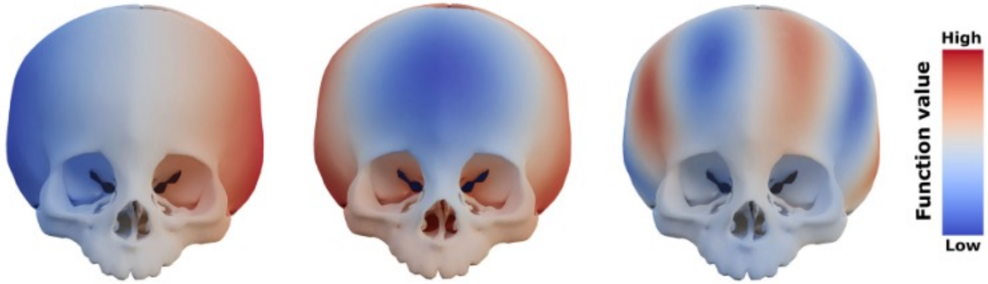


S3M: Scalable Statistical Shape Modeling through Unsupervised Correspondences

Bastian, Lennart, Alexander Baumann, Emily Hoppe, Vincent Bürgin, Ha Young Kim, Mahdi Saleh, Benjamin Busam, and Nassir Navab. "S3M: scalable statistical shape modeling through unsupervised correspondences." In *International Conference on Medical Image Computing and Computer-Assisted Intervention*, pp. 459-469. Cham: Springer Nature Switzerland, 2023.

Applications to Medical Domain

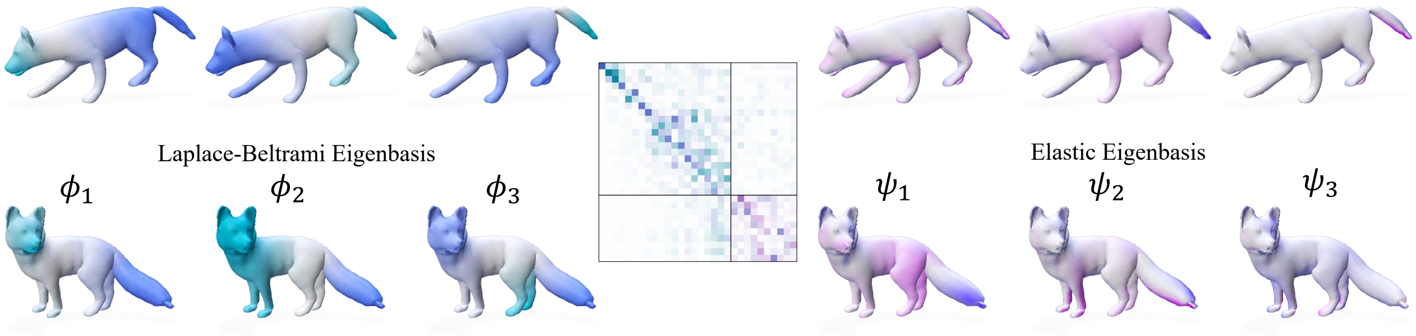




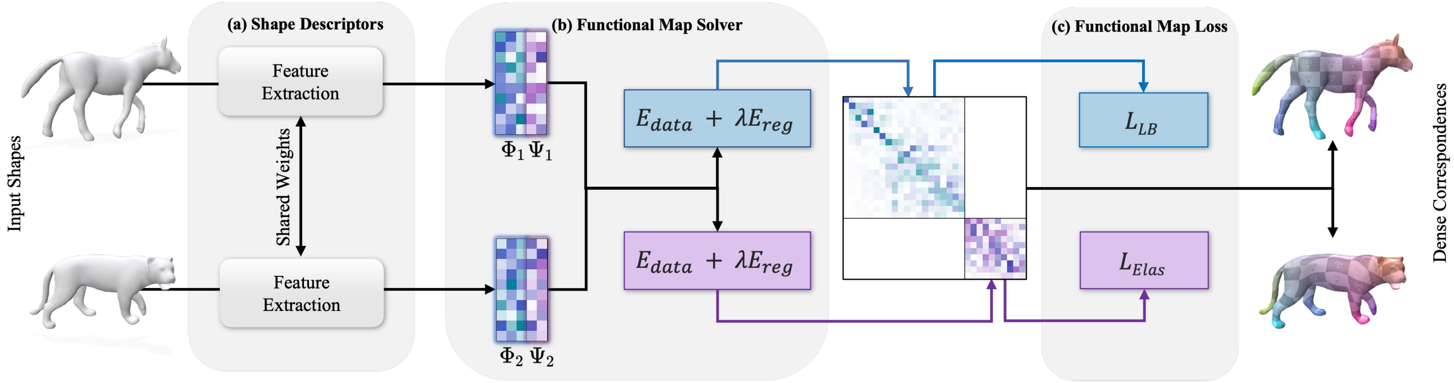
Assessing craniofacial growth and form without landmarks:
A new automatic approach based on spectral methods

Magnet, Robin, Kevin Bloch, Maxime Taverne, Simone Melzi, Maya Geoffroy, Roman H. Khonsari, and Maks Ovsjanikov. "Assessing craniofacial growth and form without landmarks: A new automatic approach based on spectral methods." *Journal of Morphology* 284, no. 8 (2023): e21609.

takeaways: that's everything



- **Hybrid Functional Maps**



- **Generalized Framework for Deep Functional Map systems with non-orthogonal basis**

Potential future directions

1. **Deformations and interpolations**
2. **More hybrid basis/Fmap (eg. learned basis, complex Fmap, …)**
3. **Partial shape matching**
4. **…**

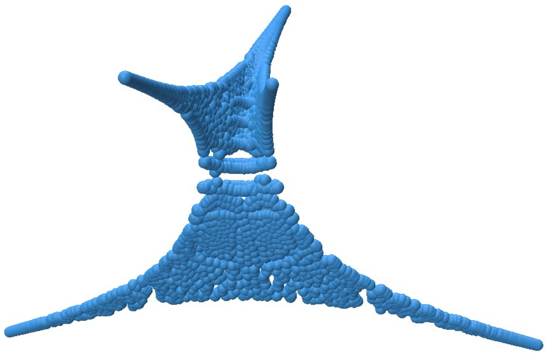
Donati, N., Corman, E., Melzi, S., & Ovsjanikov, M. (2022, February). Complex functional maps: A conformal link between tangent bundles. In *Computer Graphics Forum* (Vol. 41, No. 1, pp. 317-334).

Marin, R., Rakotosaona, M. J., Melzi, S., & Ovsjanikov, M. (2020). Correspondence learning via linearly-invariant embedding. *Advances in Neural Information Processing Systems*, *33*, 1608-1620.

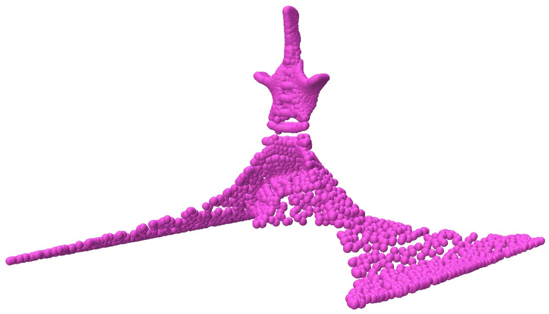
Donati, Nicolas, Etienne Corman, and Maks Ovsjanikov. "Deep orientation-aware functional maps: Tackling symmetry issues in shape matching." *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*. 2022.

Thanks!

discussions



LB first 3 Embedding



LB first 2 Embedding
+
Elas first 1 Embedding

Elas first 3 Embedding